

# Inferring Strategic Voting\*

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## Abstract

We estimate a model of strategic voting and quantify the impact it has on election outcomes. Because the model exhibits multiplicity of outcomes, we adopt a set estimator. Using Japanese general-election data, we find a large fraction [63.4%, 84.9%] of strategic voters, only a small fraction [1.4%, 4.2%] of whom *voted for a candidate other than the one they most preferred (misaligned voting)*. Existing empirical literature has not distinguished between the two, estimating misaligned voting instead of strategic voting. Accordingly, while our estimate of strategic voting is high, our estimate of misaligned voting is comparable to previous studies.

*Keywords:* strategic voting, set estimation, partially identified models, election

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# 1 Introduction

Strategic voting in elections has been of interest to researchers since Duverger (1954) and Downs (1957). Models of strategic voting are fundamental to the study of political economy, and have been used to investigate topics ranging from performance of different electoral rules to information aggregation in elections. On the other hand, there are models that take the view that voters vote sincerely according to their preferences.<sup>1</sup> Whether voters actually behave strategically, however, is an empirical question.

Strategic voting is also of interest to politicians and voters. It is widely believed that if Ralph Nader had not run in the 2000 U.S. Presidential election, Al Gore would have won the election. The presence of minor candidates and third parties affects election outcomes, and the extent of that effect depends heavily on the fraction and behavior of strategic voters.

In this paper, we study how to identify and estimate a model of strategic voting and quantify the impact strategic voting has on election outcomes by adopting an inequality-based estimator. We estimate the model using aggregate municipality level data from the Japanese general election which uses plurality rule. We then investigate what the election outcome would have been if voters voted sincerely, in our counterfactual policy experiment. Strategic voters are defined as those who make voting decisions conditioning on the event that their votes are pivotal. Unlike sincere voters who always vote according to their preferences, strategic voters do not necessarily vote for their most preferred candidate in plurality-rule elections with three or more candidates.<sup>2</sup>

In our paper, we make a clear distinction between *strategic voting*, as defined above (this is the standard definition in the theoretical literature<sup>3</sup>), and *voting for a candidate other than the one the voter most prefers* (hereafter referred to as *misaligned voting*). Strategic voters may vote for their most preferred candidate or they may not. Hence, the set of voters who engage in misaligned voting is only a subset of the set of strategic voters. Existing empirical literature has not distinguished between the two. In fact, previous attempts at estimating strategic voting have estimated misaligned voting instead of strategic voting. This distinction is important because the fraction of strategic voters is a model primitive while misaligned voting is an equilibrium object. In our paper we recover the extent of strategic voting, which allows us to conduct counterfactual policy experiments.

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<sup>1</sup>See, e.g., Palfrey (1984), Osborne and Slivinski (1996), and Callander (2005)

<sup>2</sup>There are other behavioral models of voting, such as expressive voting (voters may vote for a candidate to send a signal). We focus on sincere voting and strategic voting, which have been the main focus of the empirical literature. Accordingly, we do not attempt to quantify other types of voting, such as expressive voting, and the results in our paper depend on the two-type assumption (sincere and strategic types).

<sup>3</sup>See, e.g., the entry of “strategic voting” in *The New Palgrave Dictionary of Economics* by Feddersen (2008).

Our model is an adaptation of Myerson and Weber (1993) and Myerson (2002) with the addition of sincere voters.<sup>4</sup> We relax the equilibrium requirement that Myerson and Weber place on voters’ beliefs on pivot probabilities. We use a weaker solution concept since our identification strategy is more transparent in a model with less equilibrium restrictions. Moreover, using a weaker solution concept allows us to obtain results that are robust to different assumptions regarding voter beliefs. As we will discuss in detail later, our strategy of identifying the voters’ preferences and the fraction of strategic voters is agnostic about the equilibrium restrictions imposed between beliefs and votes. Our strategy does not depend on the particular details of the voting model, either.

Our identification argument proceeds in three steps. First we derive restrictions in terms of how preferences, which we write as a function of demographic characteristics, relate to voting behavior at the individual level. Unlike in other applications of discrete-choice models, the fact that a voter votes for candidate A does not imply that the voter preferred candidate A most. It could well be that the voter preferred candidate B over A, but voted for A instead because the voter believed that candidate B had little chance of winning. However, we can infer from the voter’s behavior that the voter did not rank candidate A last in his order of preference. It is a weakly dominated strategy for all voters, sincere and strategic, to vote for their least preferred candidate: This is how we derive restrictions that relate voter preferences to votes.

Second, we aggregate the individual-level restrictions between the votes and preferences, and relate aggregate variation in the vote shares to demographic characteristics using two particular features common to many general-election data. The first feature is that general-election data typically consists of data from many elections taking place simultaneously (e.g., 646 elections for House of Commons in U.K., 435 elections for U.S. House of Representatives).<sup>5</sup> The second feature is that the breakdown of votes and demographic characteristics *within* each electoral district is available (e.g., county-level breakdown of votes for U.S. Congressional Elections).<sup>6</sup> For the rest of the paper, we use the term “municipality” to denote the sub-district within an electoral district, such as counties. Note that several municipalities comprise one “district”, which in turn corresponds to one election (See Figure 1).

Lastly, we consider identification of the extent of strategic voting. Intuitively, the variation in the data that we would like to exploit is the variation in the voting outcome among

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<sup>4</sup>Our model can be naturally extended to elections with  $N$  candidates competing for  $N_S$  ( $N_S < N$ ) seats under single non-transferrable voting as in Cox (1994).

<sup>5</sup>As it will become clear later, we take each election to be our unit of observation.

<sup>6</sup>As we will discuss later, this data structure allows us to relate variation in the vote share to variation in the demographic characteristics within a single electoral district, holding constant common components such as beliefs over tie probabilities and candidate characteristics.

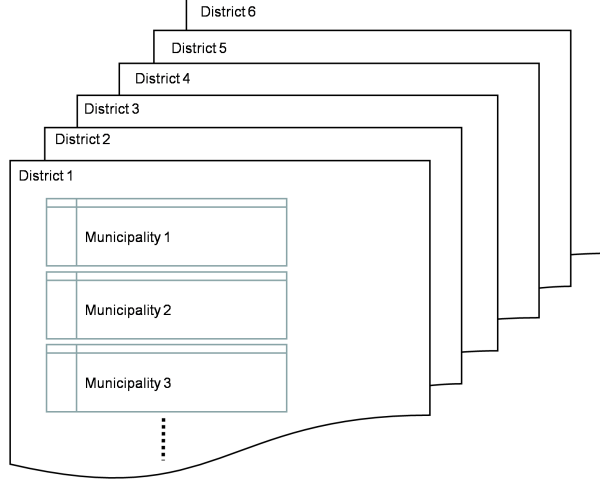


Figure 1: Data Structure. The district is our unit of observation, each of which is comprised of multiple municipalities. Breakdown of data is available at the municipality level.

*municipalities* (in different *districts*) with similar characteristics vis-à-vis the variation in the vote shares and characteristics of other municipalities in the same district. For example, consider two liberal municipalities, one in a generally conservative electoral district and the other in a generally liberal district. Suppose that there are three candidates, a liberal, a centrist and a conservative candidate in both districts. If there are no strategic voters, we would not expect the voting outcome to differ across the two municipalities. However, in the presence of strategic voters, the voting outcome in these two municipalities could differ. If the strategic voters of the municipality in the conservative district believe that the liberal candidate has little chance of winning, those voters would vote for the centrist candidate, while strategic voters in the other municipality (in the liberal district) would vote for the liberal candidate according to their preferences (if they believe that the liberal candidate has a high chance of winning).

More generally, given the preference parameters, the model can predict what the vote share would be in each municipality if all of the voters voted according to their preferences. If there were no strategic voters, the difference between the actual outcome and the predicted sincere-voting outcome would only be due to random shocks. However, when there is a large number of strategic voters, the actual vote share can systematically diverge from the predicted outcome when all voter voted sincerely. Recall that strategic voters make voting decisions conditional on the event that their votes are pivotal. If the beliefs regarding the probability of being pivotal differ across electoral districts – and we have no reason to believe that they do not – the behavior of strategic voters will also differ across districts. We can use the systematic difference between the predicted vote share and the actual vote share to

partially identify the fraction of strategic voters.

Our estimation applies an estimator based on moment inequalities developed by Pakes, Porter, Ho and Ishii (2007). We use a bounds estimator because our voting model does not yield a unique outcome and we may only be able to set-identify the model parameters.

We use data on the Japanese House of Representatives elections for estimation.<sup>7</sup> Once the primitives of the model have been estimated, we investigate the extent of misaligned voting using the estimated model. We then study how the election outcome would change if all voters voted sincerely, in our counterfactual policy experiment.

We find that a large proportion (between 63.4% and 84.9%) of voters are strategic voters. We also recover the extent of misaligned voting once we estimate the model, by simulating the equilibrium behavior. Our results show that between 1.2% and 2.7% of the voters engage in misaligned voting, or between 1.4% and 4.2% of the strategic voters.

In our counterfactual experiment, we investigate what the outcome would be if all voters vote sincerely under plurality rule. We find that the number of seats for the parties would change significantly: one party would add between 10 and 28 seats while another would lose between 17 and 39 seats out of a total of 159 seats. Even though the extent of misaligned voting is small between 1.4% and 4.2%, the impact on the number of seats is considerable because the winning margin is often small.

**Related Literature** There are both an experimental and an empirical literature on strategic voting in elections. In small-scale laboratory experiments with three candidates under plurality rule, Forsythe, Myerson, Rietz, and Weber (1993, 1996) find evidence of strategic voting.<sup>8</sup> They also find that strategic voting is more likely to occur if pre-election coordination devices such as polls and shared voting histories are available.

There is also a large empirical literature on strategic voting (see, e.g., Alvarez and Nagler (2000), Blais, Nadeau, Gidengil, and Nevitte (2001) and papers cited therein). Much of the previous work in this literature has attempted to identify strategic voting by comparing each voter’s actual vote to his preferences. Voter preferences are proxied by measures such as voting behavior in previous elections and surveys eliciting voter preferences. However, as pointed out earlier, the difference between voting and preferences is a measure of misaligned voting rather than that of strategic voting. Accordingly, our estimate of misaligned voting (between 1.2% and 2.7%) is roughly in line with the estimates of strategic voting reported

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<sup>7</sup>Our implementation does not depend on any specific institutional feature of the Japanese election. Our approach can be applied to any election with plurality rule or single non-transferrable voting.

<sup>8</sup>See Holt and Smith (2005), Morton and Williams (2006), Palfrey (2006), and Rietz (2008) for a survey of the literature on experiments.

in the previous literature, which ranges from 3% to 17%.<sup>9</sup>

More recently, Fujiwara (2011) uses regression discontinuity to study the implications of strategic voting. Using a change in the voting rule for mayoral elections at a threshold population of 200,000, he finds evidence consistent with the theory of strategic voting: Namely, that the vote share of third candidates decrease significantly in districts with plurality rule elections as opposed to districts with runoff elections.

Degan and Merlo (2007) and Myatt (2007) are two other papers that are closely related to ours. First, Degan and Merlo (2007) consider the falsifiability of sincere voting, and show that individual-level observations of voting in at least two elections are required to falsify sincere voting. They examine whether there exists a preference profile that is consistent with the observed election outcome without imposing any relationship between preferences and observable covariates. Our approach relates preferences to voter covariates within a standard discrete-choice framework. Identification of voter preferences and the fraction of strategic voters is then possible without requiring micro panel data on voting records. Our approach is analogous to papers such as Berry, Levinsohn, and Pakes (1995) which estimate individual preferences using aggregate data.<sup>10</sup>

Myatt (2007) studies strategic voting as a coordination game among a group of voters (“qualified majority”) who wish to defeat a disliked status quo. The optimal choice for the members of the qualified majority do not necessarily coincide, but they must coordinate on one choice to defeat the status quo. He shows that in equilibrium, there is some, but not full coordination among the qualified majority. He models the qualified majority as strategic voters while he models the minority as sincere voters.<sup>11</sup> He calibrates his model to the N.Y. Senatorial election in 1970 and the U.K. General Election in 1997.

Lastly, our paper is also related to the literature on strategic voter turnout.<sup>12</sup> The papers in this literature that are closest to ours are Shachar and Nalebuff (1999) and Coate, Conlin, and Moro (2008). Both papers estimate a model of voter turnout in which voter turnout is a function of the expected closeness of the election. They study turnout focusing on two candidate elections, a setting in which the issue of strategic voting does not arise. Our paper focuses on the issue of strategic voting instead of strategic turnout, although it is conceptually

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<sup>9</sup>See Alvarez and Nagler (2000), Blais, Nadeau, Gidengil, and Nevitte (2001) and papers cited therein.

<sup>10</sup>Regarding the use of aggregate data, the political science literature has been concerned about the issue of ecological inference (See, e.g., King, 1997). King (1997) proposes a solution to this problem by assuming a random coefficients type model with a particular functional form. Our approach can be thought of as microfounding the distribution of the random coefficients in his statistical model. We do so by considering a game theoretic model of voting.

<sup>11</sup>His definition of strategic voting corresponds to our definition of misaligned-voting.

<sup>12</sup>There is a large empirical literature that studies the relationship between turnout and voting. For a survey, see e.g., Blais (2006), and Merlo (2006).

straightforward to extend our approach to a model of elections with both strategic voting *and* strategic turnout. We discuss this extension at the end of Section 4.

We describe the model in the next section, and explain the data in Section 3. Details on identification and estimation are provided in Section 4. Section 5 presents the results and the counterfactual experiments. Finally, we close the paper with concluding remarks in Section 6.

## 2 Model

### 2.1 Model Set-up

Our model is an adaptation of Myerson and Weber (1993) [hereinafter denoted as MW] and Myerson (2002). We model plurality-rule elections in which  $K$  candidates compete for one seat. Voters cast a vote for one candidate,<sup>13</sup> and the candidate receiving the highest number of votes is elected to office (ties are broken with equal probability). We restrict attention to the case when  $K \geq 3$  since strategic voting is otherwise not an issue. There are  $M$  municipalities in an electoral district, and we use subscript  $m \in \{1, 2, \dots, M\}$  to denote a municipality. There are a finite number of voters,  $\sum_{m=1}^M N_m < \infty$ , who are the players of the game ( $N_m$  is the number of voters in municipality  $m$ ). Voter  $n$ 's utility from having candidate  $k$  in office is

$$u_{nk} = u(\mathbf{x}_n, \mathbf{z}_k) + \xi_{km} + \varepsilon_{nk},$$

where  $\mathbf{x}_n$  are voter characteristics,  $\mathbf{z}_k$  are candidate characteristics,  $\xi_{km}$  is a candidate-municipality shock, such as the ability of a candidate to bring pork to municipality  $m$ , and  $\varepsilon_{nk}$  is an i.i.d. preference shock.

We consider two types of voters, sincere (behavioral) and strategic (rational). A sincere voter casts his vote for the candidate he prefers most, i.e., a sincere voter votes for candidate  $k$  if and only if  $u_{nk} \geq u_{nl}, \forall l$ . On the other hand, a strategic voter casts his vote taking into consideration that the only events in which his vote is pivotal are when the election is exactly tied or when the second place candidate is one vote behind. When voter  $n$  is pivotal and he casts the decisive vote between  $k$  and  $l$ , he changes the outcome of the election. In this situation, voting for candidate  $k$  gives utility  $\frac{1}{2}(u_{nk} - u_{nl})$ .<sup>14</sup> Hence, if we let  $T_n = \{T_{n,kl}\}_{kl}$

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<sup>13</sup>We abstract from the issue of voter abstention. We discuss the issue of turnout at the end of Section 4.

<sup>14</sup>Voter  $n$ 's vote is pivotal in two cases. First, consider the case when candidates  $k$  and  $l$  are exactly tied without voter  $n$ 's vote. In this case, candidate  $k$  wins if voter  $n$  votes for  $k$ . Because ties are broken with equal probability for each candidate, the utility from voting for candidate  $k$  is  $u_{nk} - \frac{1}{2}(u_{nk} + u_{nl})$ . Second, consider the case when candidate  $k$  is one vote behind candidate  $l$  without voter  $n$ 's vote. The two candidates will tie if voter  $n$  votes for candidate  $k$ , while candidate  $l$  wins if voter  $n$  does not. Thus, the

denote voter  $n$ 's beliefs that candidates  $k$  and  $l$  will be tied for first place or that  $k$  will be one vote behind  $l$  (and assuming that  $T_{n,kl} = T_{n,lk}$ <sup>15</sup>), the expected utility from voting for candidate  $k$  is given by<sup>16</sup>

$$\bar{u}_{nk}(T_n) = \frac{1}{2} \sum_{l \in \{1, \dots, K\}} T_{n,kl}(u_{nk} - u_{nl}),$$

as in MW.

Strategic voters vote for candidate  $k$  if and only if  $\bar{u}_{nk}(T_n) \geq \bar{u}_{nl}(T_n)$ ,  $\forall l$ . Depending on the value of  $T_n$ , strategic voters may choose to vote for any candidate other than the one he prefers the least (i.e. the candidate  $k$  with the lowest value of  $u_{nk}$ ). We come back to this fact when we discuss identification.

Note that we distinguish strategic voting and misaligned voting as discussed in the Introduction. We define misaligned voting as casting a vote for a candidate other than the one the voter most prefers. Hence, only strategic voters engage in misaligned voting, but a strategic voter may or may not engage in misaligned voting. In other words, being a strategic voter is a necessary condition for misaligned voting, but not a sufficient condition.

We assume that for at least some candidate pair  $\{k, l\}$ , beliefs over pivot probability,  $T_{n,kl}$ , is non-zero. Even if there is an obvious frontrunner, there is always some chance that a vote will be pivotal although it may be very small. As long as some  $T_{n,kl}$  is always non-zero, we can normalize  $T_{n,kl}$  so that  $\sum_k \sum_{l > k} T_{n,kl} = 1$ . This normalization does not affect the voters' choices because a voter's decision is determined by the relative size of  $\bar{u}_{nk}(T_n)$ , which is not affected by rescaling  $T_{n,kl}$  by a constant factor.

We denote the type of voter  $n$  in municipality  $m$  by a random variable  $\alpha_{nm} \in \{0, 1\}$  drawn from a binomial distribution, where  $\alpha_{nm} = 0$  denotes the sincere voter and  $\alpha_{nm} = 1$  denotes the strategic voter. We also let the mean of the binomial distribution to be a random variable drawn for each municipality from some conditional distribution  $F_\alpha(\cdot | \mathbf{s}_{nk})$ . We allow the distribution  $F_\alpha$  to depend on a set of observable characteristics  $\mathbf{s}_{nk} = (w, \mathbf{x}_n, \mathbf{z}_k)$ , where  $w$  denotes election forecasts that reflect the expected closeness of the election.

Then the probability that voter  $n$  in municipality  $m$  is a strategic voter can be written as

$$\Pr(\alpha_{nm} = 1 | \alpha_m) = \alpha_m,$$

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utility from voting for  $k$  is  $\frac{1}{2}(u_{nk} + u_{nl}) - u_{nl}$ . Therefore, in both cases, the utility from voting for candidate  $k$  is  $\frac{1}{2}(u_{nk} - u_{nl})$ .

<sup>15</sup>This is a common assumption in the literature (e.g., MW and Cox, 1994), which is justified when the number of voters is not too small. Page 103 of MW explains the assumption in detail.

<sup>16</sup>We assume that voter beliefs over three-way ties are infinitesimal compared to two-way ties, as is commonly assumed in the literature.



where  $\alpha_m$  is the municipality-level random term drawn from  $F_\alpha(\cdot|\mathbf{s}_{nk})$  and we assume that  $\alpha_{nm} \perp \alpha_{n'm} \forall n, n'$  conditional on  $\alpha_m$ . The probability that the voter is sincere is  $\Pr(\alpha_{nm} = 0|\alpha_m) = 1 - \alpha_m$ .

The fraction of strategic voters may depend on the expected closeness of the race as well as other characteristics of the municipalities. For example, the fraction of strategic voters may be high when the election is expected to be close: The dependence of  $F_\alpha$  on  $w$  allows for this possibility. The reduced form way in which we incorporate this dependence avoids modeling explicitly how voters become strategic as the race becomes closer. Not modeling the dependence explicitly has the benefit that our results are robust to the exact mechanism through which some voters become strategic and others remain sincere, while simultaneously allowing us to directly estimate  $F_\alpha$  as a function of  $w$ .<sup>17</sup>

Note that explicitly microfounding the relationship between closeness and the fraction of strategic voters is akin to what Feddersen and Sandroni (2006) accomplishes with the relationship between closeness and turnout through rule utilitarianism. Unlike their paper, we do not endogenize this mechanism but rather remain agnostic about how this happens. While our reduced form way of modeling the mechanism may be somewhat unsatisfactory from a theoretical perspective, we think that it is actually beneficial from an empirical standpoint. The fact that we are treating this mechanism in a reduced form way, kind of as a black box, means that our final results are robust to the exact mechanism through which some voters become strategic and others remain sincere.

We make the following assumption on beliefs  $T_n$  following MW.

**Assumption** *Beliefs over tie probabilities  $T_n$  are common across all voters in the same electoral district, i.e.,  $T_n = T, \forall n \in \{1, \dots, N_1\} \cup \dots \cup \{1, \dots, N_M\}$ .*

This assumption simply imposes voters in the same electoral district to have common beliefs over pivot probabilities,  $T$ . For example, beliefs over pivot probabilities do not depend on the individual characteristics of the voters  $\mathbf{x}_n$  (although it may depend on the aggregate distribution of  $\mathbf{x}_n$ ).<sup>18</sup> The assumption reflects the fact that information regarding the ex-

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<sup>17</sup>Although an alternative specification would have  $F_\alpha$  depend directly on the tie beliefs  $T_n$ , we let  $F_\alpha$  depend only on  $\mathbf{s}_{nk}$ . We note that these two specifications can be partially reconciled. Because voters form beliefs using news sources as well as demographic and candidate characteristics,  $T_n$  is likely to be a function of observables  $\mathbf{s}_{nk}$  and an individual specific stochastic term,  $\eta_n$ , i.e.,  $T_n = T_n(\mathbf{s}_{nk}, \eta_n)$ . In the alternative specification with direct dependence of  $F_\alpha$  on  $T_n$ ,  $T_n = T_n(\mathbf{s}_{nk}, \eta_n)$  implies that  $F_\alpha(\cdot|T_n) = F_\alpha(\cdot|\mathbf{s}_{nk}, \eta_n)$ . Our specification can be seen as a restricted version where the dependence occurs only through observable characteristics,  $\mathbf{s}_{nk}$ , i.e.,  $F_\alpha(\cdot|\mathbf{s}_{nk})$ .

<sup>18</sup>In fact, all of our identification discussions and estimation methods go through even if the beliefs depend on an independent individual shock so long as  $T_n$  is centered around the common beliefs  $T$ . This is because when we compute the municipal level vote shares, independent individual shocks to  $T$  wash out: As a result, the municipal level vote share is only going to be a function of the common beliefs  $T$ .

pected outcome of the election is widely available from news reports and poll results. By gaining access to this kind of information, voters in the same electoral district can form similar beliefs regarding the outcome.

Let  $V_{k,m}^{SIN}$  be the fraction of votes cast by sincere voters for candidate  $k$  in municipality  $m$ , and let  $V_{k,m}^{STR}(T)$  be the fraction of votes cast by strategic voters for candidate  $k$ . Note that  $V_{k,m}^{STR}(T)$  is a function of beliefs,  $T$ . We can write these fractions as

$$V_{k,m}^{SIN} = \frac{\sum_{n=1}^{N_m} (1 - \alpha_{nm}) \cdot \mathbf{1}\{u_{nk} \geq u_{nl}, \forall l\}}{\sum_{n=1}^{N_m} (1 - \alpha_{nm})}, \quad (1)$$

$$V_{k,m}^{STR}(T) = \frac{\sum_{n=1}^{N_m} \alpha_{nm} \cdot \mathbf{1}\{\bar{u}_{nk}(T) \geq \bar{u}_{nl}(T), \forall l\}}{\sum_{n=1}^{N_m} \alpha_{nm}}. \quad (2)$$

The total vote share for candidate  $k$  in municipality  $m$  is then

$$V_{k,m}(T) = \frac{\sum_{n=1}^{N_m} (1 - \alpha_{nm})}{N_m} V_{k,m}^{SIN} + \frac{\sum_{n=1}^{N_m} \alpha_{nm}}{N_m} V_{k,m}^{STR}(T).$$

Note that these expressions are approximated by their expectation as the number of voters,  $N_m$ , becomes large, by a law of large numbers;

$$\begin{aligned} V_{k,m}^{SIN} &\xrightarrow{p} v_{k,m}^{SIN} \equiv \iint \mathbf{1}\{u_{nk} \geq u_{nl}, \forall l\} g(\boldsymbol{\varepsilon}) d\boldsymbol{\varepsilon} f_m(\mathbf{x}) d\mathbf{x}, \text{ and} \\ V_{k,m}^{STR}(T) &\xrightarrow{p} v_{k,m}^{STR}(T) \equiv \iint \mathbf{1}\{\bar{u}_{nk}(T) \geq \bar{u}_{nl}(T), \forall l\} g(\boldsymbol{\varepsilon}) d\boldsymbol{\varepsilon} f_m(\mathbf{x}) d\mathbf{x}, \end{aligned}$$

where  $f_m$  denotes the distribution of the demographic characteristics,  $\mathbf{x}$ , in municipality  $m$ , and  $g$  denotes the distribution of idiosyncratic shocks,  $\boldsymbol{\varepsilon}_n = (\varepsilon_{n1}, \dots, \varepsilon_{nK})$ . We obtain these expressions by computing the vote share for candidate  $k$  among voters of a given demographic characteristics  $\mathbf{x}$ , and then integrating this vote share with respect to characteristics  $\mathbf{x}$  using its distribution  $f_m$ . We obtain a similar expression for the total vote share as  $N_m$  becomes large:

$$V_{k,m}(T) \xrightarrow{p} v_{k,m}(T) \equiv (1 - \alpha_m) v_{k,m}^{SIN} + \alpha_m v_{k,m}^{STR}(T). \quad (3)$$

## 2.2 Solution Outcome

Until now, our model has been the same as the one considered in MW with the only difference being the presence of sincere voters. While MW proceeds by imposing equilibrium restrictions on voters' beliefs to obtain sharp predictions on the outcome, it turns out that

for our empirical purposes, we can greatly relax their equilibrium restrictions. Below, we explain our solution concept, compare it with the equilibrium of MW, and provide a discussion of the reason why we use our solution concept.

Let us denote the district level vote share, which is the total number of votes obtained by a candidate divided by the total number of votes cast in the election, by  $V_k \equiv \sum_{m=1}^M N_m V_{k,m} / \sum_{m=1}^M N_m$ . MW imposes the following consistency requirement in equilibrium:  $V_k > V_l \Rightarrow \varepsilon T_{kj} \geq T_{lj}$ ,  $\forall \varepsilon \in [0, 1)$ ,  $\forall k, l, j$ . This implies that pivot probabilities involving candidates with low vote shares are zero. The first consistency requirement (**C1**) we impose on beliefs is a much weaker version of MW's ordering condition:

**C1** : For an election with  $K$  candidates,

$$V_k > V_l \Rightarrow T_{kj} \geq T_{lj} \quad \forall k, l, j \in \{1, \dots, K\}.$$

This condition simply implies that pivot probabilities involving candidates with high vote shares are larger than those with low vote shares. For the case of  $K = 3$  with vote shares  $V_1 > V_2 > V_3$ , **C1** implies that  $T_{12} \geq T_{13} \geq T_{23}$ , i.e., beliefs on the pivot probability between candidates 1 and 2,  $T_{12}$ , is higher than those between candidates 1 and 3,  $T_{13}$ , and so on. Note that the restrictions on beliefs imposed under the equilibrium of MW upon observing  $V_k > V_l$  is order of magnitude more stringent than as imposed under **C1**.

Our second condition, **C2**, simply requires that given beliefs  $T$ , strategic voters vote optimally (and sincere voters vote for their most preferred candidate).

**C2** : For candidate  $k$  in municipality  $m$ ,

$$V_{k,m} = \frac{\sum_{n=1}^{N_m} (1 - \alpha_{nm})}{N_m} V_{k,m}^{SIN} + \frac{\sum_{n=1}^{N_m} \alpha_{nm}}{N_m} V_{k,m}^{STR}(T).$$

Now we define the solution outcome of the voting game.

**Definition** A set of solution outcomes  $W \subseteq \Delta^{K C_2} \times (\times_{m=1}^M \Delta^K)$  is defined as the set  $W = \left\{ T, \left\{ \{V_{k,m}\}_{k=1}^K \right\}_{m=1}^M \right\}$  such that **C1** and **C2** are satisfied.

A few comments are in order. We begin with a discussion of why we use this solution concept instead of that of MW. Previewing our identification strategy, we propose to identify the fraction of strategic voters in a way that *does not rely on particular equilibrium restrictions on beliefs*. The basic identification idea, which we will fully describe in Section 4, is that we can use the variation in the vote share of, say, liberal municipalities in liberal districts vis-a-vis liberal municipalities in conservative districts to bound the share of strategic voters. Note that this idea can be implemented without relying on specific equilibrium restrictions

imposed between beliefs and votes: In particular, this idea is not specific to the Myerson-Weber equilibrium, and it can possibly be implemented with other equilibrium concepts. We choose our solution concept because it is both simple, and allows us to implement the idea using actual data. Proceeding in this manner also has the benefit of being robust to alternative specifications.

Now, we discuss some of the properties of our set of solution concept. First, the set of solution outcomes,  $W$ , is not empty: That is, a solution outcome exists. This can be shown in a similar way as in the proof of Theorem 1 in MW. The proof is in Appendix A. Second,  $W$  is not a singleton in general. In order to cope with the issue of multiplicity of solution outcomes, we adopt an inequality-based estimator in our estimation. Third,  $W$  is a superset of the set of equilibria considered in MW. This is because condition **C1** is weaker than that of MW.<sup>19</sup> Lastly, note that  $W$  does not depend on the information structure of the model, i.e., whether we assume that the voters know the realization  $\alpha_{nm}$  and  $\varepsilon_{nk}$  of other voters, or only their distributions.

Finally, we remark on the empirical restriction implied by our solution outcome.<sup>20</sup> Note that **C2** embodies the restriction that no voter votes for his least preferred candidate through equations (1) and (2), which give the expressions for vote shares of the sincere and strategic voters. However, beyond this restriction, the model leaves considerable freedom in how  $V_{k,m}^{STR}(T)$  is linked to voter preferences. This is because the solution outcome does not pin down  $T$  (only a weak restriction is imposed via **C1**), nor do we observe the value of  $T$ .<sup>21</sup>

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<sup>19</sup>The fact that **C1** is weaker than MW means that the beliefs satisfying **C1** includes the rational expectations beliefs of MW, but can also include other beliefs.

<sup>20</sup>We briefly discuss the empirical restrictions imposed by the original equilibrium of MW and compare it to our solution outcome, which is more flexible and can better account for the variation in the data. The equilibrium of MW predicts that either (i) the first place candidate wins, and the second and third place candidates receive exactly the same number of votes (with corresponding beliefs  $\{T_{12}, T_{13}, T_{23}\} = \{p, 1-p, 0\}$  for some  $p \in [0, 1]$ ) or (ii) the third place candidate receives zero votes (with beliefs  $\{T_{12}, T_{13}, T_{23}\} = \{1, 0, 0\}$ ). Even if we (1) introduce sincere voters, (2) add shocks to voter preferences or (3) introduce randomness to the fraction of strategic voters (or any combination of (1), (2), and (3)) to MW, there would still only be two types of equilibria: One with beliefs  $\{T_{12}, T_{13}, T_{23}\} = \{p, 1-p, 0\}$  and the other with  $\{T_{12}, T_{13}, T_{23}\} = \{1, 0, 0\}$ . Equilibrium (i) still has the undesirable property that the second and third candidates receive exactly the same number of votes. In Equilibrium (ii), all three candidates can receive a positive and different number of votes. However, this type of equilibria cannot generate elections where  $T_{12}, T_{13}, T_{23}$  are all positive (Equilibrium (i) also cannot generate such elections as well.) There are some observations in our data set that ended up being a very close three-way race, where  $T_{12}, T_{13}, T_{23}$  were clearly all positive. Furthermore, there are many borderline cases which make it difficult for the econometrician to determine whether imposing the beliefs  $\{T_{12}, T_{13}, T_{23}\} = \{1, 0, 0\}$  is appropriate. Because we adopt a weaker solution outcome (which contains all of the MW equilibria), we can proceed without imposing the strong and sometimes inappropriate restrictions.

<sup>21</sup>To the extent that we do not impose restrictions on the beliefs,  $T$ , and only require that voting decision be best response to some  $T$ , the empirical content of our solution outcome would be similar if we had instead adopted rationalizability as our solution concept (See Bernheim, 1984, Pearce, 1984).

### 3 Data

We use data from the Japanese House of Representatives election held on September 11, 2005. Out of a total number of 480 Representatives, 300 members were elected by plurality rule. We use the data from these 300 plurality-rule elections.<sup>22</sup> For each electoral district, the breakdown of vote-share data is available by municipality as shown in Figure 1. An electoral district is usually comprised of several municipalities (9.23 on average, in our sample).<sup>23</sup> This particular data structure plays an important role in our identification.

We obtained the data on the vote shares and candidate characteristics from *Yomiuri Shimbun*, a national newspaper publisher and *Asahi-Todai Elite Survey 2005 (ATES)*. The *ATES* is a survey of candidates with regard to their policy positions on various issues.<sup>24</sup> We construct a measure of candidates' ideology using this survey.<sup>25</sup> The demographic characteristics we use are obtained from the *Social and Demographic Statistics of Japan* published by the Statistics Bureau of the Japanese Ministry of Internal Affairs and Communications.<sup>26</sup> Data on pre-election forecasts are collected from two periodicals, *Shukan Asahi* and *Shukan Gendai*. They have district-by-district election forecasts, which we use as a measure of the expected closeness of the election,  $w$ .

Out of a total of 300 districts, we keep the districts that satisfy the following criteria.

- (i) There are three or four candidates,<sup>27</sup> and the composition of the candidates' parties in the district is any three or four of the following four parties; the Liberal Democratic

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<sup>22</sup> An additional 180 Representatives were elected by proportional representation from 11 regional electoral districts. In proportional representation, voters cast ballots for parties, and a closed list is used to determine the winner. It is possible for a person to be a candidate in both plurality and proportional elections. When two candidates are ranked equally on the party list, the results of the plurality rule election affect the relative rank of the two candidates. Only the LDP and the DPJ ranked more than two candidates equally in this election.

<sup>23</sup> In the vast majority of cases, municipal borders do not cross electoral districts.

<sup>24</sup> This survey was conducted by the labs of Ikuo Kabashima and Masaki Taniguchi of the Faculty of Law and Political Science, University of Tokyo and the *Asahi Shimbun*.

<sup>25</sup> Since there is heterogeneity in ideology even among members of the same party (see for e.g. Nemoto et. al (2008)), it would be ideal if we can construct a measure of politician' ideology from actual roll call votes as in Poole and Rosenthal (1997). We, however, cannot use such data because party discipline is strongly enforced in the Japanese Diet and there is little variation in the roll call vote within a given party. Also, a significant fraction of candidates has not held any public office before the election. For these reasons, we rely on the survey data.

<sup>26</sup> The basic information for the data is available at <http://www.stat.go.jp/english/data/ssds/outline.htm> and <http://www.stat.go.jp/english/data/zensho/intex.html>.

<sup>27</sup> We do not include 15 observations in which there are only two candidates for technical reasons. We use an estimator of Pakes, Porter, Ho and Ishii (2007) in our estimation, but it is not clear whether their method of inference can be applied when some of the parameters are point-identified and others are only set-identified. While two candidate districts contain no information about the extent of strategic voting (since all voters, both strategic and sincere, vote according to their preferences), they point-identify some of the preference parameters of the voters. For our estimation, this is problematic. Alternatively, we can use other inequality based estimators (e.g. Chernozhkov, Hong and Tamer (2007)), which give consistent

Party (LDP), the Democratic Party of Japan (DPJ), the Japan Communist Party (JCP), or the Yusei (YUS). Technically, the Yusei is not a single party, but we grouped former LDP candidates who split away from the LDP and ran on a common platform against postal privatization.

(ii) There are at least two municipalities within the electoral district, and no demographic data is missing at the municipality level.

(iii) There are no mergers of municipalities within the electoral district during the period from April 1, 2004 to the day of the election.

(iv) Responses to *ATES* are available for all candidates.

We are left with 159 electoral districts. We drop samples that do not satisfy criterion (i) because we treat party affiliation as a candidate characteristic, and we cannot precisely estimate the coefficients on parties that only fielded a very small number of candidates. Criterion (i) ensures that we have enough elections with the same combination of parties fielding candidates to construct our moment inequalities.<sup>28</sup> We need criterion (ii) because our estimation requires at least two municipalities in each electoral district. Criterion (iii) is required to deal with an issue that arises when merging two data sets. Because the demographics data and the vote share data are collected on different dates (April 1, 2004 and September 11, 2005), municipalities that merged with others between these dates are dropped from the sample. In some cases, however, we are able to match the data properly. When this is possible, we keep the merging municipalities in the sample.

We report the descriptive statistics of electoral-district vote shares in Table 1. There are 9.23 municipalities per electoral district on average. The average winner’s vote share is about 52% and the winning margin is about 14%. The mean vote share of the winner is higher in three-candidate districts (52.9%) than in four-candidate districts (40.5%). The mean winning margin is also higher in three-candidate districts (14.1%) than in four-candidate districts (8.5%). Similarly, the margin between the second- and third-place candidates is significantly lower in four-candidate districts than in three-candidate districts. Pre-election forecasts on closeness are reported in the next three rows of Table 1. The closeness measure is in intervals of 0.5 and a value of 1 corresponds to the closest and a value of 4 corresponds to the least close.<sup>29</sup> The next four rows report the vote-share breakdown for the four political

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estimates even when a subset of the parameters are point identified. However, this comes at a very high computational cost in our application.

<sup>28</sup>The Kagoshima 5th District is dropped from the sample because no other district had the same combination of parties fielding candidates (LDP, JCP, YUS) as this district. This is the only district we dropped that satisfied all three criteria.

<sup>29</sup>The two periodicals report on each election and each of the elections falls into one of four categories: (1)

	mean	st. dev.	min	max	# obs
# of municipalities per district	9.23	7.27	2	36	159
3-candidate district	8.72	7.03	2	36	144
4-candidate district	14.13	8.02	3	36	15
winner's vote share (%)	51.72	6.83	28.98	73.62	159
3-candidate district	52.90	5.70	36.03	73.62	144
4-candidate district	40.46	6.69	28.98	55.89	15
winning margin (%)	13.53	10.23	0.06	53.92	159
3-candidate district	14.05	10.17	0.17	53.92	144
4-candidate district	8.50	9.73	0.06	35.50	15
margin between 2nd and 3rd (%)	28.51	9.67	0.00	43.32	159
3-candidate district	30.39	7.65	0.00	43.32	144
4-candidate district	10.45	8.51	0.57	23.32	15
pre-election forecast on closeness	2.33	0.81	1	4	159
3-candidate district	2.36	0.82	1	4	144
4-candidate district	2.07	0.59	1.5	3.5	15
vote share – JCP	7.62	2.72	2.77	17.02	154
vote share – DPJ	38.56	8.80	10.78	60.10	159
vote share – LDP	49.66	8.90	23.19	73.62	159
vote share – YUS	34.95	9.10	14.50	49.58	20
ideology – JCP	1.97	0.36	1	2.75	154
ideology – DPJ	3.10	0.60	1	4.50	159
ideology – LDP	3.12	0.61	1.25	4.67	159
ideology – YUS	2.55	0.45	1.25	3.25	20

Table 1: Descriptive Statistics of Electoral Districts – Vote Shares

parties. The mean vote share of the LDP is 49.7%, the highest among all parties. It is followed by the DPJ with 38.6%, the YUS with 35.0% and the JCP with 7.6%.<sup>30</sup>

The last four rows of Table 1 report candidate's economic ideology by party. The measure of ideology is constructed from candidate responses to questions regarding economic policy in *ATES* and takes a value between 1 and 5, where a larger value corresponds to pro-market ideology and *vice versa*.<sup>31</sup> Because party affiliation of candidates captures most of

a race that is neck and neck (2) a race with a slightly leading candidate (3) a race with a likely winner (4) a race with a clear winner. We construct the closeness measure by assigning a value of 1 to the first category, 2 to the second category, etc, then take the average of the two periodicals.

<sup>30</sup>Note that the sum of these percentages is greater than 100%. This is because not all parties field candidates in every district.

<sup>31</sup>We use five questions asked in *ATES* regarding the candidate's position on economic ideology such as how much they agree with the statement, "the size of government should be small." We take the average of the responses to the five questions. We acknowledge that to the extent that the survey data does not capture candidate ideology perfectly, our estimates may suffer from attenuation bias.

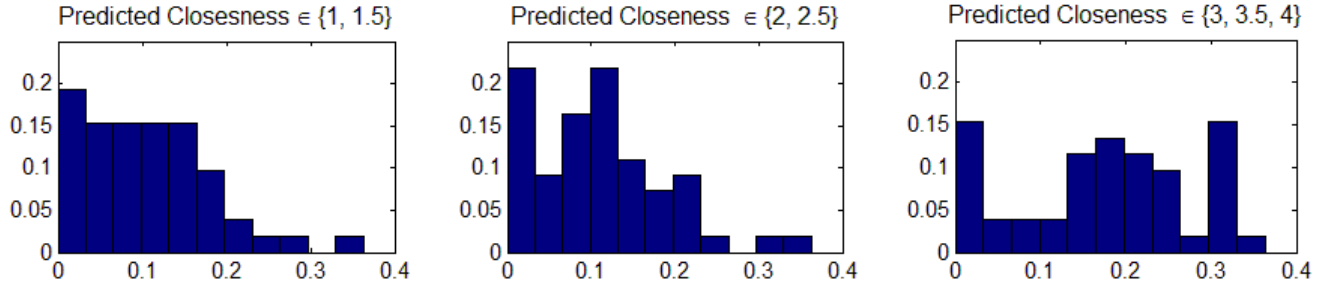


Figure 2: Histogram of the Winning Margin by Predicted Closeness. The vertical axis corresponds to the frequency and the horizontal axis is the winning margin.

the variation in responses to questions concerning political ideology, we use survey responses related to economic ideology.<sup>32</sup>

Figure 2 is a histogram of the winning margin by predicted closeness of elections. The vertical axis corresponds to the frequency and the horizontal axis is the winning margin. The first panel is the histogram for elections that were predicted to be close, with the measure of predicted closeness equal to  $\{1, 1.5\}$ . The second panel corresponds to those with measures equal to  $\{2, 2.5\}$ , and the third panel corresponds to those predicted to have a clear winner, with measures between 3 and 4. These panels show that when the elections are predicted to be close, the winning margin tends to be small.

Table 2 reports the descriptive statistics of candidate characteristics. The first three rows contain information on the candidates' hometowns.<sup>33</sup> The next three rows provide descriptive statistics on the candidates' political experience. An average of 1.32 (in three-candidate districts) and 1.47 (in four-candidate districts) candidates are incumbents. Note that the number of incumbents is higher than 1 because some candidates who had previously been elected to the House of Representatives in a proportional-rule election ran in the plurality election. Less than 0.51 candidates on average have previously held public office.<sup>34</sup>

Table 3 reports the descriptive statistics of the municipalities' demographic characteristics. The mean income per capita is about 3.16 million yen (about \$35,000), and the mean length of schooling is about 12 years on average. The mean fraction of the population above

<sup>32</sup>For example, there is zero variation in survey responses to questions related to political ideology among candidates of the JCP.

<sup>33</sup>In case a candidate has a hometown in his/her electoral district (as reported in the first row), we have additional information on candidates' hometowns that identifies exactly which municipality the candidate's hometown is in. We do not report it here, but use it in our estimation.

<sup>34</sup>This includes former and current municipality councillors, mayors, members of a prefectural assembly, prefectural governors, and the Members of the Houses of Councillors, as well as former Members of the House of Representatives.



	3 cand. district	4 cand. district
# of candidates w/ hometown in district	1.01 (0.96)	1.71 (1.05)
# of candidates w/ hometown in prefecture	0.95 (0.86)	0.71 (0.92)
# of candidates w/ hometown in another pref.	1.04 (0.82)	1.58 (1.23)
# of incumbents	1.32 (0.53)	1.47 (0.51)
# of candidates who previously held public office	0.51 (0.62)	0.35 (0.49)
# of candidates with no exp. in public office	1.16 (0.67)	2.18 (0.73)
# of observations	158	17

Table 2: Descriptive Statistics of Electoral Districts – Candidate Characteristics. The mean of each variable is reported. Standard errors are in parenthesis.

	mean	st. dev.	min	max	# obs
income per capita (in million yen)	3.16	0.42	2.27	6.47	1,621
years of schooling $\leq 11$ years (%)	35.00	12.37	7.16	71.08	1,621
12-14 years (%)	45.41	6.37	20.09	62.59	1,621
15-16 years (%)	9.83	3.34	2.86	19.41	1,621
$\geq 16$ years (%)	9.76	5.86	1.51	39.38	1,621
population above age 65 (%)	22.45	7.16	8.06	49.71	1,621

Table 3: Descriptive Statistics of Municipalities

age 65 is 22.5 percent. In the estimation, we use the distribution of demographic characteristics, which is readily available for years of schooling and age. Regarding income, only the mean of the distribution was available at the municipality level. We use the prefectural Gini coefficients as well as the average income to construct the distribution.<sup>35</sup>

<sup>35</sup>We have data on the total taxable income and the total number of taxpayers for each municipality. The mean income for each municipality can be computed from these numbers. We compute the quantiles of the income distribution by assuming a log-normal distribution where the variance is calculated by fitting the prefecture-level income distribution. Data on the prefecture-level income distribution is obtained from *the 2004 National Survey of Family Income and Expenditure* published by the Statistics Bureau of the Japanese Ministry of Internal Affairs and Communications.

## 4 Identification and Estimation

We first describe the econometric specification of the model we have presented in Section 2 in order to facilitate our identification and estimation arguments. Then, we discuss the identification and the estimation of the model.

### 4.1 Specification

We specify the utility function of voter  $n$  in municipality  $m$  with candidate  $k$  elected to office as

$$u_{nmk} = u(\mathbf{x}_n, \mathbf{z}_{km}; \theta^{PREF}) + \xi_{km} + \varepsilon_{nk},$$

where  $\xi_{km}$  is an i.i.d. idiosyncratic candidate-municipality level shock which follows a normal distribution,  $N(0, \theta_\xi)$ , denoted as  $F_\xi$ , and  $\varepsilon_{nk}$  is an i.i.d. idiosyncratic voter-candidate level shock which follows a Type-I extreme value distribution. An example of  $\xi_{km}$  is the candidate's ability to bring pork spending to municipality  $m$ .  $\theta^{PREF}$  is a vector of preference parameters.  $\mathbf{x}_n$  denotes the characteristics of voter  $n$ , including years of education, income level, and an indicator of whether or not the voter is above age 65.  $\mathbf{z}_{km} = \{\mathbf{z}_k^{POS}, \mathbf{z}_{km}^{QLTY}\}$  is a vector of observable attributes of candidate  $k$  in municipality  $m$ . We partition  $\mathbf{z}_{km}$  depending on how it interacts with voter characteristics. Let  $\mathbf{z}_k^{POS}$  be the attributes of candidate  $k$  which are related to his ideological position such as his party affiliation and *ATES* score regarding economic ideology. Let  $\mathbf{z}_{km}^{QLTY}$  be other non-ideological attributes of candidate  $k$  such as the candidate's previous political experience and an indicator of whether municipality  $m$  is the candidate's hometown (which is why  $\mathbf{z}_{km}$  is indexed by  $m$ ).

As for  $u(\mathbf{x}_n, \mathbf{z}_{km}; \theta^{PREF})$ , we assume the following functional form

$$u(\mathbf{x}_n, \mathbf{z}_{km}; \theta^{PREF}) = -\|\theta^{ID} \mathbf{x}_n - \theta^{POS} \mathbf{z}_k^{POS}\| + \theta^{QLTY} \mathbf{z}_{km}^{QLTY},$$

where  $\theta^{PREF} = \{\theta^{ID}, \theta^{POS}, \theta^{QLTY}\}$ . The first term of  $u(\cdot)$  is the (dis)utility of electing a candidate whose ideal policy is different from the voter. We write this as a function of the distance between the ideological position of the voter,  $\theta^{ID} \mathbf{x}_n$ , and the position of the candidate  $\theta^{POS} \mathbf{z}_k^{POS}$ . Both  $\theta^{ID} \mathbf{x}_n$  and  $\theta^{POS} \mathbf{z}_k^{POS}$  are two dimensional vectors which we write as linear functions of the voter's demographics,  $\theta^{ID} \mathbf{x}_n$ , and candidate characteristics  $\theta^{POS} \mathbf{z}_k^{POS}$ . The first dimension of  $\theta^{ID} \mathbf{x}_n$  and  $\theta^{POS} \mathbf{z}_k^{POS}$  is political ideology and the second dimension is economic ideology.<sup>36</sup> The last term,  $\theta^{QLTY} \mathbf{z}_{km}^{QLTY}$ , captures the non-ideological

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<sup>36</sup>The variables in  $\mathbf{z}_k^{POS}$  that determine economic ideology are the survey responses from *ATES* which we explained in Section 3. *ATES* also includes questions that are related to what can be described as political ideology. As we mentioned earlier, however, the party affiliation of the candidate is a very good proxy for

component of utility.<sup>37</sup>

As described in the model section, the objective of a sincere voter is to vote for candidate  $k$ , who gives the highest value of  $u_{nmk}$ , while the objective of a strategic voter is to vote for candidate  $k$ , who gives the highest value of  $\bar{u}_{nmk}(T)$ , where  $\bar{u}_{nmk}(T)$  is defined as follows:

$$\bar{u}_{nmk}(T) = \sum_{l \in \{1, \dots, K\}} T_{kl}(u_{nmk} - u_{nml}).$$

As we discussed in Section 2, we assume that for at least some candidate pair  $\{k, l\}$ ,  $T_{kl}$  is positive, no matter how small. This allows us to normalize  $T$  so that  $\sum_k \sum_{l > k} T_{kl} = 1$ , because utility representation is invariant to multiplication by a constant factor.

Recall that we denote the type of voter  $n$  in municipality  $m$  by a random variable  $\alpha_{nm} \in \{0, 1\}$  drawn from a binomial distribution, where  $\alpha_{nm} = 0$  denotes the sincere voter and  $\alpha_{nm} = 1$  denotes the strategic voter. Then the probability that voter  $n$  in municipality  $m$  is a strategic voter can be written as

$$\Pr(\alpha_{nm} = 1 | \alpha_m) = \alpha_m.$$

We let  $\alpha_m$ , the mean of the binomial distribution, to be a random variable which is drawn from  $F_\alpha(\cdot | \mathbf{s}_{nk})$  for each municipality. We let  $F_\alpha(\cdot)$  be a function of  $\mathbf{s}_{nk}$  to allow the fraction of strategic voters to depend on the expected closeness of the race as well as other characteristics of the municipalities. It may be the case, for example, that the fraction of strategic voters is higher when the election is expected to be closer. We specify  $F_\alpha(\cdot | \mathbf{s}_{nk})$  as a Beta distribution  $Beta(\theta_{\alpha 1}(\mathbf{s}_{nk}), \theta_{\alpha 2}(\mathbf{s}_{nk}))$ .

## 4.2 Identification

In this subsection, we discuss the identification of the model when we let the number of districts (denoted as  $D$ ) go to infinity. As described in the Data Section, our election data includes observations from many districts, for each of which we have a municipality-level breakdown of vote-share data and demographic characteristics. In terms of our notation, the number of districts is large ( $D \rightarrow \infty$ ), but the number of municipalities per electoral district, denoted by  $M^d$ , is small ( $M^d < \infty, \forall d \in \{1, \dots, D\}$ ). We assume that voting games

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the survey reponse to these questions. For this reason we use the party dummy instead of the ATES surevey responses as determinants of political ideology. Hence an alternative interpretation of our specification is that this term captures average party ideology.

<sup>37</sup> Although the functional form we introduce here is commonly used in the literature, we cannot rule out other possible functional forms. While our identification argument does not rely on the particular functional form, our estimation does impose these functional forms.

(i.e., elections) are played in  $D$  districts independently of each other, and we treat each district as a unit of observation.

Our identification argument proceeds in two steps. We first discuss partial identification of preference parameters. Then, given partial identification of preference parameters, we discuss partial identification of the fraction of strategic voters.

#### 4.2.1 Partial Identification of Preference Parameters

Preference parameters are (partially) identified by the relationship between demographic and vote-share variation within each electoral district that we observe in the data. In order to exploit this variation for identification of preference parameters, the main restriction we use is that voters do not vote for their least preferred candidate. We augment this restriction with the common belief assumption. We illustrate below how the two restrictions combine to identify voter preferences.

Consider a municipality in which the distribution of preference orderings over Candidates A, B, and C are as shown in Figure 3. We first consider the restriction imposed on the vote shares by the fact that voters do not vote for the least preferred candidate. This is shown as Restriction (I) in Figure 3. Given the distribution of preference orderings in the municipality, this restriction implies that the vote share for each candidate should be in  $[0, 2/3]$ . The reason is as follows. Take the vote share of Candidate A,  $V_A$ , for example. We can bound  $V_A$  above by  $2/3$ , since voters whose preferences are  $B \succ C \succ A$  and  $C \succ B \succ A$  do not vote for Candidate A. On the other hand, we can only bound  $V_A$  below by 0, because even voters with preferences  $A \succ B \succ C$  and  $A \succ C \succ B$  may vote for Candidate B and Candidate C respectively, if the beliefs over tie probabilities involving Candidate A is 0 ( $T_{AB} = T_{AC} = 0$ ).

Next, consider the restriction imposed on the vote shares by the common belief assumption. This is shown as Restriction (II) in Figure 4. Now, we can no longer have voters whose preferences are  $B \succ A \succ C$  and  $C \succ A \succ B$  to vote for Candidate A at the same time, unlike in the previous case. In order to have voters with preference ordering  $B \succ A \succ C$  to vote for Candidate A, we must have  $T_{AC}$  be close to 1, and  $T_{AB}$ ,  $T_{BC}$  close to 0 (and  $\alpha_m \simeq 1$ ).<sup>38</sup> On the other hand, in order to have voters with preference ordering  $C \succ A \succ B$  to vote for Candidate A, we must have  $T_{AB}$  close to 1, and  $T_{AC}$  and  $T_{BC}$  close to 0 (and  $\alpha_m \simeq 1$ ). These two beliefs cannot coincide if we impose the common belief assumption. Hence, we obtain a tighter upper bound on  $V_A$ .

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<sup>38</sup>Note that the fraction of strategic voters ( $\alpha_m$ ) affects the bound on the vote shares. The smaller the  $\alpha_m$ , the tighter the restriction on voter shares. For example, if  $\alpha_m = 0$ , then we have point prediction, i.e.  $V_A = V_B = V_C = 1/3$ . As we wish to obtain a bound that holds for any value of  $\alpha_m$ , the bound on the vote shares illustrated in Figure 3 corresponds to the case of  $\alpha_m \simeq 1$ . This is the case which gives the largest bound (The bound for all other values of  $\alpha_m$  is smaller and contained inside this bound.).

Ordering	Pr( Ordering   $\theta$ )	Restriction (I)	Restriction (II)
$A \succ B \succ C$	$\dots$ 1/6	$V_A \in [0, 2/3]$	$V_A \in [0, 1/2]$
$A \succ C \succ B$	$\dots$ 1/6	$V_B \in [0, 2/3]$	$V_B \in [0, 1/2]$
$B \succ A \succ C$	$\dots$ 1/6	$V_C \in [0, 2/3]$	$V_C \in [0, 1/2]$
$B \succ C \succ A$	$\dots$ 1/6		
$C \succ A \succ B$	$\dots$ 1/6		
$C \succ B \succ A$	$\dots$ 1/6		

Figure 3: Identification of Preferences. Restriction (I) corresponds to the bounds on the vote shares we obtain when we just rely on the fact that voters do not vote for the least preferred candidate. Restriction (II) corresponds to the bounds when we impose common beliefs within the municipality.

The common belief assumption does have some identification power as we saw in the previous paragraph, but so far the restriction on the vote shares may not appear so strong. Recall, however, that we impose common beliefs not just within a municipality but across municipalities as well. This adds an extra restriction on the vote shares.

Consider a district with two municipalities  $m_1$  and  $m_2$  with distribution of preferences as shown in Figure 4. Following the previous discussion, the vote shares of all three candidates are bounded by  $[0, 1/2]$  in  $m_1$  and by  $[0, 1/4]$  for Candidate A and  $[0, 3/4]$  for Candidates B and C in  $m_2$  (This is shown next to the preference ordering for each municipality). Suppose we observe vote shares  $(V_A, V_B, V_C) = (0, 1/2, 1/2)$  in  $m_1$  and  $(1/4, 3/4, 0)$  in  $m_2$ . Taken independently, these vote shares are consistent with the distribution of preference orderings. However, there is no belief that can rationalize these vote shares jointly. (We require  $(T_{AB}, T_{AC}, T_{BC}) \simeq (0, 0, 1)$  in order to justify the vote share in  $m_1$  while we require  $(T_{AB}, T_{AC}, T_{BC}) \simeq (1, 0, 0)$  in order to justify the vote share in  $m_2$ .<sup>39</sup>) Hence, with common beliefs across municipalities, the distribution of preference ordering as illustrated in Figure 4 can rule out such vote share pairs. In other words, if we observe such vote share pairs, the distribution of preference orderings cannot be as shown in Figure 4. We note that the identification power of common beliefs strengthens as the number of municipalities within districts increases.

We emphasize that our previous discussion does not depend on knowing the fraction

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<sup>39</sup>More precisely, these restrictions on the vote shares are obtained when beliefs  $T$  are as described in the text and  $\alpha_m \simeq 1$ , the case which gives us the largest bounds. See the previous footnote for details.

Ordering in $m_1$		Ordering in $m_2$	
$A \succ B \succ C \dots 1/6$		$A \succ B \succ C \dots 0$	
$A \succ C \succ B \dots 1/6$		$A \succ C \succ B \dots 0$	
$B \succ A \succ C \dots 1/6$	$V_A^1 \in [0, 1/2]$	$B \succ A \succ C \dots 1/4$	$V_A^2 \in [0, 1/4]$
$B \succ C \succ A \dots 1/6$	$V_B^1 \in [0, 1/2]$	$B \succ C \succ A \dots 1/4$	$V_B^2 \in [0, 3/4]$
$C \succ A \succ B \dots 1/6$	$V_C^1 \in [0, 1/2]$	$C \succ A \succ B \dots 1/4$	$V_C^2 \in [0, 3/4]$
$C \succ B \succ A \dots 1/6$		$C \succ B \succ A \dots 1/4$	

$$(V_A^1, V_B^1, V_C^1) = (0, 1/2, 1/2)$$

$$(V_A^2, V_B^2, V_C^2) = (1/4, 3/4, 0)$$

Figure 4: Identification of Preference. The restrictions on the vote shares become tighter as we impose common beliefs across municipalities (within the same district).

of strategic voters or the beliefs in each district. Consider the following exercise: Suppose we have a preference parameter that rationalizes the observed data (implying that this parameter value is in the identified set). Suppose next that we change the value of the preference parameter to a new value which increases the mean utility that voters get from Candidate A. By appropriately changing the fraction of  $\alpha$ , and/or beliefs  $T$ , we may be able to keep the vote shares for all candidates unchanged. Then this means that the new preference parameter value is also in the identified set. On the other hand, if we cannot find any  $\alpha$  or  $T$  for the new parameter value then the new value is not in the identified set. Because we search over all possible values<sup>40</sup> of  $\alpha$  and  $T$  (rather than using a particular value for  $a$  and  $T$ ) to rationalize the vote shares for each value of the preference parameter, our identification argument does not depend on knowing the values of  $\alpha$  or  $T$ .

Finally, we rephrase our identification argument more formally using model notation. Note that the exact vote shares across municipalities are determined by  $T^d$  (beliefs over tie probabilities in district  $d$ ) and  $\{\alpha_m\}_{m=1}^{M_d}$  (the fraction of strategic voters in each municipality) neither of which are observed by the econometrician. Because neither  $T^d$  nor  $\{\alpha_m\}_{m=1}^{M_d}$  are observed, we can only bound the set of vote shares that is consistent for each preference parameter  $\theta^{REF}$ . This set can be obtained by fixing  $\theta^{REF}$ , and varying  $a_m \in [0, 1]$  and  $T^d \in \Delta^{K_{C_2}}$  that satisfy the consistency requirement **C1**.<sup>41</sup> If the actual observed vote shares lie in this set, then  $\theta^{REF}$  is in the identified set and vice versa. The identified set of

<sup>40</sup>Subject to the consistency requirement on beliefs **C1**.

<sup>41</sup>In fact, we obtain the largest bounds when  $\{\alpha_m\}_{m=1}^{M_d}$  are set to 1 as discussed in Footnote 39. Hence, in order to obtain this set of vote shares, we fix  $\{\alpha_m\}_{m=1}^{M_d}$  to 1 and just vary  $T^d$ .

preference parameters is then the union of all  $\theta^{PREF}$  for which there exists a  $T^d \in \Delta^{KC_2}$  that satisfy **C1** and it rationalizes the observed data.

#### 4.2.2 Partial Identification of the Fraction of Strategic Voters

Second, we discuss the identification of the average fraction of strategic voters. In the following discussion, we fix the preference parameters,  $\theta^{PREF}$ , and consider the identification of the extent of strategic voting given  $\theta^{PREF}$ . Once this is accomplished, we can vary  $\theta^{PREF}$  in the identified set of  $\theta^{PREF}$  to trace out the identified set of the parameters that determine the extent of strategic voting.

Intuitively, the variation in the data that we would like to exploit is the variation in the voting outcome among *municipalities* (in different *districts*) with similar characteristics vis-à-vis the variation in the vote shares and characteristics of other municipalities in the same district. For example, consider two districts, one that is generally conservative and another that is liberal. Suppose that we can find a liberal municipality from each district. Suppose also that there are three candidates, a liberal, a centrist and a conservative candidate in both districts. If there are no strategic voters, we would not expect the voting outcome to differ across the two municipalities. However, in the presence of strategic voters, the voting outcome in these two municipalities could differ. If the strategic voters of the municipality in the conservative district believe that the liberal candidate has little chance of winning, those voters would vote for the centrist candidate, while the strategic voters in the other municipality (in the liberal district) would vote for the liberal candidate according to their preferences (if they believe that the liberal candidate has a high chance of winning).

More generally, given the preference parameters, the model can predict what the vote share would be in each municipality if all of the voters voted according to their preferences. If there were no strategic voters, the difference between the actual outcome and the predicted sincere-voting outcome would only be due to random shocks. However, when there is a large number of strategic voters, the actual vote share can systematically diverge from the predicted outcome. This is due to the multiplicity of solution outcomes induced by strategic voters. Recall that strategic voters make voting decisions conditional on the event that their votes are pivotal. If the beliefs regarding the probability of being pivotal differ across electoral districts – and we have no reason to believe that they do not – the behavior of strategic voters will also differ across districts. This corresponds to different outcomes being played in different districts. We use the systematic difference between the predicted vote share and the actual vote share to partially identify the fraction of strategic voters.

To further illustrate our identification argument, consider the case of three candidates. In this case, the vote shares in municipality  $m$  can be drawn as a point in a simplex. Recall

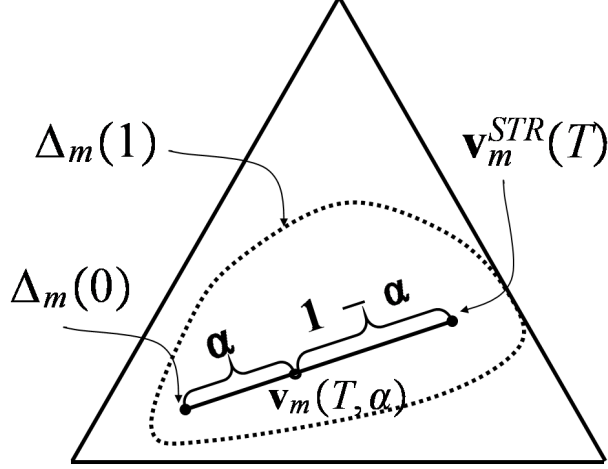


Figure 5: Vote Shares for the Case of  $N = 3$ . Vote shares  $\mathbf{v}_m(T, \alpha)$  is a mixture of sincere votes ( $\Delta_m(0) = \mathbf{v}_m^{SIN}$ ) with fraction  $1 - \alpha$ , and strategic votes ( $\mathbf{v}_m^{STR}(T)$ ) with fraction  $\alpha$ .

that given a particular value of  $\alpha_m$  (the fraction of strategic voters in municipality  $m$ ) and  $T$ , the vote shares can be written as a convex combination of the vote shares of sincere and strategic voters;

$$\mathbf{v}_m(T, \alpha_m) = (1 - \alpha_m)\mathbf{v}_m^{SIN} + \alpha_m\mathbf{v}_m^{STR}(T).$$

where  $\mathbf{v}_m$  is the vector of vote shares of the three candidates  $(v_{1m}, v_{2m}, v_{3m})$  and similarly for  $\mathbf{v}_m^{SIN}$  and  $\mathbf{v}_m^{STR}$ . Notice that here, we have made the dependence of  $\mathbf{v}_m$  on  $\alpha_m$  explicit. Now define  $\Delta_m(\alpha_m)$  as the set of all possible vote shares when we vary  $T$  in  $\mathfrak{T}$  (We denote the set of  $T$  satisfying **C1** by  $\mathfrak{T}$ ),

$$\Delta_m(\alpha_m) = \bigcup_{T \in \mathfrak{T}} \mathbf{v}_m(T, \alpha_m).$$

Note that  $\Delta_m(\alpha_m)$  and  $\Delta_m(1)$  are similar, by a factor of  $\alpha_m$  around the singleton  $\Delta_m(0) = \mathbf{v}_m^{SIN}$  because  $\alpha_m$  is the weight of the convex combination. The dotted circle in Figure 5 corresponds to  $\Delta_m(1)$ .

For expositional purposes, we first present our identification argument when we can take the number of municipalities to go to infinity and the municipality level shock  $\xi_m$  is close to zero. Consider a subset of municipalities in a single electoral district which all have the same demographic characteristics (Note that this does not literally have to be the case because we can control for demographic characteristics once preference parameters are known). In this case, the vote share observations should all lie on the line segment between  $\Delta_m(0) = \mathbf{v}_m^{SIN}$



and  $\mathbf{v}_m^{STR}(T)$  because these two endpoints are the same in all municipalities<sup>42</sup> and only the realizations of  $\alpha_m$  vary across municipalities. Denote the support of this empirical distribution as  $L$  and the endpoints of  $L$  as  $\underline{L}$  and  $\bar{L}$ . We also define the point  $L'$  where the extension of  $L$  intersects the boundary  $\Delta_m(1)$  (See Figure 6). If we were able to tell the exact position of  $\mathbf{v}_m^{STR}(T)$  on  $L$ , we can fully identify the distribution of  $\alpha$  by this empirical distribution as we will describe more below.

As a basis for our following discussion, we begin by clarifying what is identified just from the observations and what is not. First,  $\Delta_m(0)$  is identified once preferences are identified. Next, the line segment  $L$  is identified because  $L$  just corresponds to the support of the empirical distribution of the vote shares. Correspondingly, the end points of  $L$ ,  $\underline{L}$  and  $L'$ , are also identified. On the contrary, the location of  $\mathbf{v}_m^{STR}(T)$ , i.e., the point that corresponds to the vote share among strategic voters, is not identified. The reason is that  $\mathbf{v}_m^{STR}(T)$  depends on  $T$ , which we do not know. However, the following observation gives us a bound on the location of  $\mathbf{v}_m^{STR}(T)$ . Recall that  $\mathbf{v}_m$  is a convex combination of  $\Delta_m(0)$  and  $\mathbf{v}_m^{STR}(T)$  as  $\mathbf{v}_m(T, \alpha_m) = (1 - \alpha_m)\mathbf{v}_m^{SIN} + \alpha_m\mathbf{v}_m^{STR}(T)$  for some  $\alpha \in [0, 1]$ . This fact implies that  $\mathbf{v}_m^{STR}(T)$  must be located somewhere between  $\bar{L}$  and  $L'$ .

Now, consider two polar cases, Case A and Case B in Figure 6. Case A depicts the situation where  $\mathbf{v}_m^{STR}(T)$  is at  $\bar{L}$  and Case B depicts the situation where  $\mathbf{v}_m^{STR}(T)$  is at  $L'$ . For each of the two cases, observations of vote shares can be mapped into realizations of  $\alpha_m \in [0, 1]$ . This mapping is different in Case A and Case B and results in different distributions of  $\alpha$  as can be seen in Figure 6. Case A corresponds to the upper bound of the extent of strategic voting, and Case B provides the lower bound. We therefore can partially identify the distribution of  $\alpha_m$  as well as the upper and lower bounds of its mean.

Now we discuss how we can modify this discussion to the case where the number of municipalities are finite but the number of districts goes to infinity. Parallel to the previous argument, consider subsets of municipalities from each district with the same demographic characteristics. The key differences from the previous situation are that (1) even if we condition on the same demographics,  $\mathbf{v}_m^{STR}(T)$  differs across districts because  $T$  is not the same across districts, and (2) we can only take a finite number of municipalities from the same district. Figure 7 illustrates the case where we have three municipalities from two districts. Notice that  $\Delta_m(0)$  is the same across these municipalities because the demographics are the same. However, as municipalities in different districts have different  $T^d$ , the vote share data will be on different line segments for different districts. As in the previous argument,

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<sup>42</sup>To see this, recall that  $\Delta_m(0)$  is a function of demographic characteristics, and  $\mathbf{v}_m^{STR}(T)$  is a function of demographic characteristics and  $T$ . As the municipalities belong to the same district they share the same  $T$  and they share the same demographic characteristics because of the way in which we selected them.

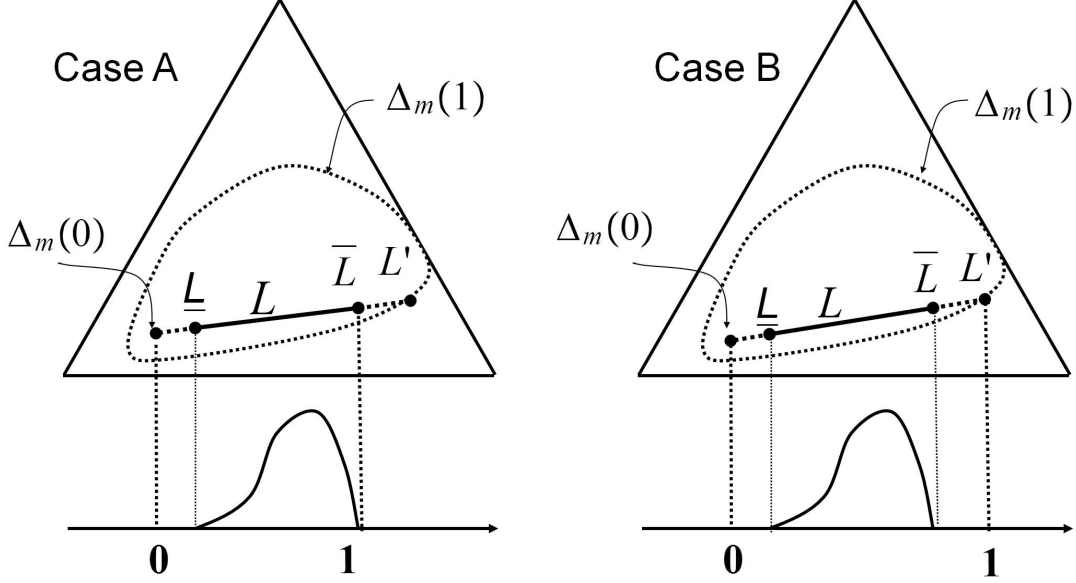


Figure 6: Partial Identification of the Extent of Strategic Voting When  $D = 1$  and  $M^d \rightarrow \infty$ . Vote share observations map differently into different values of  $\alpha$  depending on the position of  $v_m^{STR}(T)$ . Case A corresponds to the upper bound of the distribution ( $\bar{L}$  corresponds to  $\alpha = 1$ ), and Case B to the lower bound ( $L'$  corresponds to  $\alpha = 1$ ).

consider two polar cases, Case A' and Case B' in Figure 7. Case A' is the situation where  $\mathbf{v}_m^{STR}(T)$  is at  $\bar{L}_m$  and Case B' corresponds to the situation where  $\mathbf{v}_m^{STR}(T)$  is at  $L'_m$ . For each of the two cases, we can map the vote share observations into realization of  $\alpha_m \in [0, 1]$ . Note that even though the number of municipalities in a given district is finite, by taking the number of districts to infinity, we can obtain an infinite number of  $\alpha_m$  on  $[0, 1]$  that are transformed from the vote share observations. Note that Case A' gives the upper bound of the distribution of  $\alpha_m$ , and Case B' gives the lower bound. Thus, we set-identify the distribution of  $\alpha_m$ .

In the actual data, the vote shares may not lie on the same line segment as in Figure 7, even when we take observations from municipalities with the same demographics. Recall that  $\xi_m$  is the municipality level shock that accounts for this kind of variation. It is true that if we do not restrict the distribution of  $\xi_m$  in any way, it may not be possible to separately identify the distribution of  $\xi_m$  and  $\alpha_m$  nonparametrically. However, it should be intuitive from Figure 7 that if restrict the distribution of  $\xi_m$  to well-behaved distributions which are mean-zero and unimodal, the same intuition would carry through. We assume that the distribution of the random shock  $\xi_m$  follows a Normal distribution with mean zero. Then, we can parametrically account for the dispersion of vote shares around the line segment and the above identification discussion remains valid.

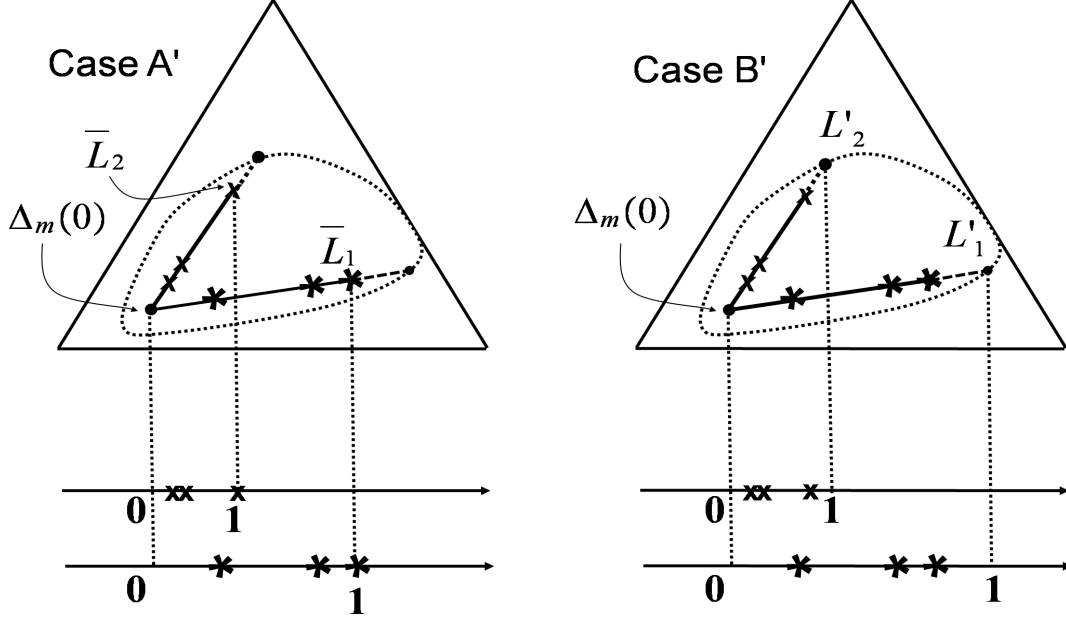


Figure 7: Partial Identification of the Extent of Strategic Voting When  $D \rightarrow \infty$ , but  $M^d < \infty$ . The figure illustrates the situation when there are two districts with three municipalities each. Case A' corresponds to the upper bound, and Case B' to the lower bound.

So far, our discussion in this section has mostly centered around identification of the distribution of  $\alpha_m$  within the context of our model. How does this discussion relate to our intuitive identification argument that we gave at the beginning of this section? As before, consider two *districts*, one that is generally liberal and another that is conservative. Suppose that we can find a liberal *municipality* from each district, say  $m_{LL}$  and  $m_{CL}$ . Now, the vote shares in municipality  $m_{LL}$ ,  $v_{LL}$ , will be located close to  $\Delta_m(0)$ . The vote shares  $v_{CL}$  in municipality  $m_{CL}$ , on the other hand, may not be close to  $\Delta_m(0)$ . Then we see that misaligned voting must have been at least as big as  $|v_{CL} - v_{LL}|$ . Since misaligned voting is a subset of strategic voting, we know that strategic voting must be bigger than  $|v_{CL} - v_{LL}|$ . This is how we obtain the lower bound of strategic voting. How do we obtain the upper bound? Suppose we can take a second liberal municipality,  $m'_{CL}$  from the same district as  $m_{CL}$ . The voters in  $m'_{CL}$  have the same incentives to vote strategically as the voters in  $m_{CL}$ , but suppose that we observe  $|v'_{CL} - v_{LL}| < \frac{1}{2} |v_{CL} - v_{LL}|$ , for example. Then the fraction of strategic voters has to be less than  $\frac{1}{2}$  in municipality  $m'_{CL}$  (i.e.,  $\alpha_{m'_{CL}} < 1/2$ ) This is how we obtain the upper bound. Note that if we could only find one municipality in each district, Case A' will always imply  $\alpha_m = 1$  and we would only get a trivial upper bound on strategic voters. To conclude, the distribution of the extent of strategic voting is identified from the

different degrees to which voters in municipalities such as  $m_{CL}$  and  $m'_{CL}$  (it is important that we have multiple municipalities) vote differently from voters in  $m_{LL}$ .

Finally we describe how to extend our argument when preference parameters are only partially identified. For each  $\theta^{REF}$  in the identified set, we can partially identify the extent of strategic voting by following our previous argument. To the extent that preference parameters are only partially identified, we can vary  $\theta^{REF}$  in the identified set: This allows us to trace out the identified set of the extent of strategic voting.

### 4.3 Estimation

At the outset, it is useful to clarify the set of parameters that we estimate: They are the preference parameters,  $\theta^{REF}$ , the distribution of strategic voters,  $(\theta_{\alpha 1}, \theta_{\alpha 2})$ , and the variance of  $\xi$ ,  $\theta_{\xi}$ . It is important to note that we do not estimate the beliefs  $T$ . This is because our unit of observation is the district, and as the number of districts increases, so does the number of tie beliefs  $T$ . Because we cannot treat  $T$  as parameters, we need restrictions that do not involve  $T$ .

We estimate the model using an inequality-based estimator developed by Pakes, Porter, Ho, and Ishii (2007). If voter beliefs,  $T$ , were known (either observed, or uniquely determined by the model), a single outcome would correspond to one realization of the unobserved error terms  $(\xi, \alpha)$ . In such a case, we could employ estimation procedures such as GMM or MLE. However, as discussed in Section 4.2, the multiplicity of outcomes induced by the presence of strategic voters, together with the fact that we cannot observe voter beliefs,  $T$ , imply that the model parameters are only partially identified: This makes the use of a set-based estimator appropriate.

We construct the moment inequalities using an idea which is somewhat similar to indirect inference (Smith (1993) and Gouriéroux, Monfort, and Renault (1993)). The following explains the steps we take to construct the moment inequalities. A more detailed description of each step is found in Appendix B.

1. Take some district  $d$  and denote the municipalities that belong to this district as  $\{1, 2, \dots, M^d\}$ . Regress the vote share data of candidate  $k$  in each municipality,  $v_{k,m}^{data}$ , on the demographics of each municipality,  $f_m$ ,<sup>43</sup> to obtain the regression coefficient

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<sup>43</sup>We used  $f_m$  to denote the distribution of demographic characteristics  $\mathbf{x}$  in municipality  $m$  in Section 2. If we discretize the distribution, we can identify  $f_m$  with a vector of probabilities that assign a probability to each discretized bin. We use the same notation  $f_m$  here to denote the vector of probabilities. The vector  $f_m$  contains, for example, the fraction of the population above 65, the fraction of population in different income ranges, etc. Note that we cannot take the number of regressors (the number of elements in  $f_m$ ) to be too large because the number of municipalities in each district is small and finite. Rather than including

$\beta_{k,d}^{data} = (f_d' f_d)^{-1} f_d' v_{k,d}^{data}$ , where  $v_{k,d}^{data} = (v_{k,1}^{data}, \dots, v_{k,M^d}^{data})'$  and  $f_d = (f_1, \dots, f_{M^d})'$ .<sup>44</sup>

2. Fix some parameter  $\theta$  and beliefs of voters,  $T^d$ . Also fix particular values of  $\alpha_d = \{\alpha_m\}_{m=1}^{M^d}$  and  $\xi_d = \{\xi_m\}_{m=1}^{M^d}$ , which are the fractions of strategic voters and the candidate-municipality shocks, respectively. Given  $\theta$ ,  $T^d$ ,  $\alpha_d$  and  $\xi_d$ , compute the predicted vote share outcome for each municipality in the district,  $(v_{k,1}^{PRED}(T^d, \alpha_1, \xi_1; \theta), \dots, v_{k,M^d}^{PRED}(T^d, \alpha_{M^d}, \xi_{M^d}; \theta))$ .
3. Parallel to step 1, regress the simulated vote share,  $v_{k,m}^{PRED}(T^d, \alpha_m, \xi_m; \theta)$ , on the demographic characteristics in each municipality,  $f_m$ , to obtain the regression coefficient  $\beta_{k,d}(T^d, \alpha_d, \xi_d; \theta) = (f_d' f_d)^{-1} f_d' v_d^{PRED}(T^d)$ , where  $v_d^{PRED}(T^d) = (v_{k,1}^{PRED}(T^d, \alpha_1, \xi_1; \theta), \dots, v_{k,M^d}^{PRED}(T^d, \alpha_{M^d}, \xi_{M^d}; \theta))'$ .
4. Because we do not know  $T^d$ , we vary  $T^d \in \mathfrak{T}(v_d^{data})$  to obtain the element-by-element minimum and maximum values of the regression coefficients as

$$\begin{aligned} \underline{\beta}_{k,d}(\alpha_d, \xi_d; \theta) &= \min_{T^d \in \mathfrak{T}(v_d^{data})} \beta_{k,d}(T^d, \alpha_d, \xi_d; \theta), \text{ and} \\ \bar{\beta}_{k,d}(\alpha_d, \xi_d; \theta) &= \max_{T^d \in \mathfrak{T}(v_d^{data})} \beta_{k,d}(T^d, \alpha_d, \xi_d; \theta), \end{aligned}$$

where  $v_d^{data} = \left( \sum_{m=1}^{M^d} v_{k,m}^{data} N_m / \sum_{m=1}^{M^d} N_m \right)_{k=1}^{K^d}$  is the *district* level vote share data and  $\mathfrak{T}(v_d^{data})$  is defined as the set of beliefs that is consistent with condition **C1**. Recall that **C1** requires that beliefs be consistent with the vote share outcome.

5. Integrate out  $\alpha_d$  and  $\xi_d$  by simulating values of  $\alpha_d$  and  $\xi_d$  from  $F_\alpha$  and  $F_\xi$ , and obtain  $\bar{\beta}_{k,d}(\theta) = \iint \bar{\beta}_{k,d}(\alpha_d, \xi_d; \theta) dF_\alpha dF_\xi$  and  $\underline{\beta}_{k,d}(\theta) = \iint \underline{\beta}_{k,d}(\alpha_d, \xi_d; \theta) dF_\alpha dF_\xi$ .
6. Then, by construction, we have  $E[\underline{\beta}_{k,d}(\theta_0)] \leq E[\beta_{k,d}^{data}] \leq E[\bar{\beta}_{k,d}(\theta_0)]$  at the true parameter  $\theta_0$ . Thus, we obtain the following moment inequalities;

$$\begin{aligned} E[\underline{\beta}_{k,d}(\theta_0) - \beta_{k,d}^{data}] &\leq 0, \text{ and} \\ E[\bar{\beta}_{k,d}(\theta_0) - \beta_{k,d}^{data}] &\geq 0. \end{aligned}$$

Moreover, we can construct moment inequalities conditioning on candidate character-

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all the relevant demographic characteristics in  $f_m$  and running a single regression, what we do instead is to run multiple univariate regressions, each corresponding to a subset of the regressors. We explain this in Appendix B in more detail.

<sup>44</sup>We run separate regressions for each of  $K - 1$  candidates. Because the vote shares add up to one, we omit the regression for one of the candidates.

istics  $z$  ( $z$  only takes discrete values).<sup>45</sup> We can do so by running the regressions in steps 1 and 3 only on a subset of the sample for which candidate characteristics  $z$  takes a particular value:

$$\begin{aligned} E[\underline{\beta}_{k,d}(\theta_0) - \beta_{k,d}^{data} | z] &\leq 0, \text{ and} \\ E[\bar{\beta}_{k,d}(\theta_0) - \beta_{k,d}^{data} | z] &\geq 0. \end{aligned}$$

The identified set is the set of  $\theta$  that satisfy the above equations.

We base our estimation on the conditional moment inequalities. We take the sample analog of the conditional moment inequalities by repeating steps 1 through 5 for each district. Then, by taking the average, we obtain the criterion function

$$\begin{aligned} Q^+(\theta) &= \sum_{\zeta,k} \left\| \frac{1}{D} \sum_d 1_{\{z=\zeta\}} [\bar{\beta}_{k,d}(\theta) - \beta_{k,d}^{data}] \right\|_{-}, \\ Q^-(\theta) &= \sum_{\zeta,k} \left\| \frac{1}{D} \sum_d 1_{\{z=\zeta\}} [\beta_{k,d}^{data} - \underline{\beta}_{k,d}(\theta)] \right\|_{+}, \end{aligned}$$

where  $\|a\|_{+} = \max\{0, a\}$ , and  $\|a\|_{-} = \min\{0, a\}$ . We then apply Pakes, Porter, Ho, and Ishii (2007).

Note that in computing the predicted vote shares in Step 3, we use  $v_{k,m}(T)$  in equation (3).  $v_{k,m}(T)$  is the infinite counterpart of the vote share  $V_{k,m}(T)$  in equation (3); that is, the probability limit of  $V_{k,m}(T)$  when the number of voters tends to infinity. Of course, the number of voters in each municipality is finite,<sup>46</sup> but this is not a problem as long as the error from approximating the vote share by its infinite counterpart is sufficiently small compared to the variance of other error terms in the model.

**Extending the Model to Include Voter Turnout** Our approach can be extended to include the voter's turnout decision. We can, for example, introduce a cost of voting (or a consumption value of voting) into our model, and allow the voters to abstain. In terms of the standard discrete choice model, this would be analogous to the inclusion of an outside option (e.g., not buying a good). Of course, with this modification, we would no longer be able to normalize  $T$  to sum up to 1 (i.e.,  $\sum_k \sum_{l>k} T_{kl} = 1$ ) as we do in our paper. The scale of  $T$  matters for turnout. However, it should be straightforward in principle to identify

<sup>45</sup>  $z$  only includes variables such as indicators for party affiliation and hometown as described in Section 4.1.

<sup>46</sup> The average number of voters in a municipality is more than 43,000.

and estimate a model with voter turnout. The scale of  $T$  would be identified by the level of turnout. Then, the identification of the model parameters would follow similarly as the discussion in Section 4.2. The actual estimation would proceed by simulating the vote shares and turnout for all possible values of  $T$  including those that do not add up to 1.

In this paper, we only focus on the issue of strategic voting for computational reasons. In the standard pivotal voter model, turnout is sensitive to small changes in  $T$ . For example, a change in  $T$  from  $10^{-11}$  to  $10^{-10}$  increases the voter's benefit of turning out by ten-fold. This means that we would need to simulate the outcome on a grid in the space of pivot probability that is fine enough to clearly differentiate values  $10^{-11}$ ,  $10^{-10}$  (and in between). Hence, the computational cost of implementing this approach could be very high.

We present our parameter estimates in the next section with the caveat that we have not dealt with the issue of voter turnout. We note that this may result in an underestimate of the set of strategic voters as strategic voters may strategically abstain more frequently than sincere voters.

## 5 Results and Counterfactual Experiment

### 5.1 Parameter Estimates

The 95% confidence intervals for the parameters are calculated following Pakes, Porter, Ho, and Ishii (2007), and are reported in Table 4. The 95% confidence intervals reported in Table 4 are also consistent estimates of the identified set for each dimension. The exact specification of the utility function we estimate is

$$\begin{aligned}
u(\mathbf{x}_n, \mathbf{z}_{km}; \theta^{PREF}) = & \\
& - \{ [\theta_1^{const}, \theta_1^{income}, \theta_1^{education}, \theta_1^{above65}, \theta_1^{below65}] \mathbf{x}_n - [\theta^{LDP}, \theta^{JCP}, \theta^{DPJ}, \theta^{YUS}] \mathbf{z}_{k,1}^{POS} \}^2 \\
& - \{ [\theta_2^{const}, \theta_2^{income}, \theta_2^{education}, \theta_2^{above65}, \theta_2^{below65}] \mathbf{x}_n - \theta^{ATES} z_{k,2}^{POS} \}^2 \\
& + [\theta^{incumbent}, \theta^{previous}, \theta^{no\_experience}, \theta^{hometown1}, \theta^{hometown2}, \theta^{hometown3}, \theta^{hometown4}] \mathbf{z}_{km}^{QLTY} \\
& + \xi_{km} + \varepsilon_{kn},
\end{aligned}$$

where  $\mathbf{z}_{k,1}^{POS}$  is a vector of political party indicator variables,  $z_{k,2}^{POS}$  denotes candidate's *ATES* score, and we use normalizations  $\theta_1^{below65} = \theta_2^{below65} = 0$ ,  $\theta^{no\_experience} = 0$ ,  $\theta^{hometown4} = 0$ , and  $\theta^{LDP} = 0$ .<sup>47</sup> As for the distribution of  $\alpha$ , we specified  $F_\alpha(\cdot | \mathbf{s}_{nk})$  to be a Beta distribution

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<sup>47</sup>If we let the first three elements of the vector  $\mathbf{z}_{km}^{QLTY}$  be dummy variables for whether (1) candidate  $k$  has been an incumbent, (2) has had previous political experience, or (3) has had no political experience, then the first three elements of  $\mathbf{z}_{km}^{QLTY}$  add up to 1:  $z_{km}^{QLTY}(1) + z_{km}^{QLTY}(2) + z_{km}^{QLTY}(3) = 1$ . Thus we need to

Parameter	Confidence Interval	Parameter	Confidence Interval
$\theta_1^{const}$	$[-0.556, -0.543]$	$\theta^{ATES}$	$[0.268, 0.279]$
$\theta_1^{income}$	$[-0.028, -0.025]$	$\theta^{incumbet}$	$[0.159, 0.174]$
$\theta_1^{educatoin}$	$[-0.022, -0.021]$	$\theta^{previous}$	$[-0.312, -0.303]$
$\theta_1^{above65}$	$[0.136, 0.141]$	$\theta^{hometown1}$	$[0.784, 0.793]$
$\theta^{YUS}$	$[-0.701, -0.695]$	$\theta^{hometown2}$	$[-0.069, -0.058]$
$\theta^{JCP}$	$[-2.495, -2.482]$	$\theta^{hometown3}$	$[0.088, 0.096]$
$\theta^{DPJ}$	$[-1.975, -1.969]$	$\theta_\xi$	$[1.810, 1.836]$
$\theta_2^{const}$	$[2.629, 2.635]$	$\theta_{\alpha 1}^{const}$	$[1.841, 1.860]$
$\theta_2^{income}$	$[-0.637, -0.625]$	$\theta_{\alpha 2}^{const}$	$[0.240, 0.241]$
$\theta_2^{education}$	$[0.068, 0.070]$	$\theta_{\alpha 1}^{closeness}$	$[-0.148, -0.135]$
$\theta_2^{above65}$	$[-0.056, -0.052]$	$\theta_{\alpha 2}^{closeness}$	$[0.066, 0.120]$

Table 4: Confidence Intervals. Confidence intervals reported are asymptotically more conservative than 95%. These confidence intervals are calculated following Pakes, Porter, Ho, and Ishii (2007).

with parameters  $\theta_{\alpha 1}(\cdot)$  and  $\theta_{\alpha 2}(\cdot)$  where  $\theta_{\alpha 1}(\cdot)$  and  $\theta_{\alpha 2}(\cdot)$  are estimated to be a function of  $w$ ,

$$\begin{aligned}\theta_{\alpha 1}(w) &= \theta_{\alpha 1}^{const} + \theta_{\alpha 1}^{closeness} w \\ \theta_{\alpha 2}(w) &= \theta_{\alpha 2}^{const} + \theta_{\alpha 2}^{closeness} w.\end{aligned}$$

We allowed  $F_\alpha$  to depend on  $w$  in order to let the fraction of strategic voters be potentially correlated with pre-election forecasts.

First, we discuss our parameter estimates for the first and second terms of the utility function. These terms capture the ideological component of the voter’s utility; the first term considers the general congruence in terms of political ideology between the voter and the candidate’s party, and the second term captures the ideological affinity between the voter and the individual candidate regarding economic policy that is not fully captured by the first term. Both terms are written as functions of the distance between the voter’s and the candidate’s ideological positions. The estimated ideological positions of the candidate’s parties are  $\theta^{JCP} = [-2.495, -2.482]$ ,  $\theta^{DPJ} = [-1.975, -1.969]$ , and  $\theta^{YUS} = [-0.701, -0.695]$ , where  $\theta^{LDP} = 0$ , by normalization. We can interpret this result as follows. The JCP and the DPJ

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normalize one of the coefficients (The fact that we are dealing with a discrete choice model precludes us from including a constant term as well.). For the same reason,  $\theta^{below65}$  and  $\theta^{hometown4}$  are normalized to 0. As for  $\theta^{LDP}$ , this is normalized to 0 because only the difference between the candidate’s ideology,  $\theta^{POS} \mathbf{z}_k^{POS}$ , and the voter’s ideology,  $\theta^{ID} \mathbf{x}_n$  matter. Note that because we include a constant term in  $\mathbf{z}_k^{POS}$ , one of the elements in  $\theta^{ID}$  can be normalized to zero.



	JCP	DPJ	LDP	YUS
Vote Share				
Actual	7.6	38.6	49.7	34.9
Effect of $\Delta(\text{Income})$	[0.03, 0.95]	[-0.21, 0.78]	[-1.37, 0.01]	[-0.09, 0.56]
Effect of $\Delta(\text{Schooling})$	[-0.04, 0.32]	[0.09, 2.18]	[-2.18, -0.01]	[-1.34, -0.48]
Effect of $\Delta(\text{Age} > 65)$	[-0.02, -0.01]	[-0.85, -0.03]	[0.03, 0.95]	[0.08, 0.33]
Number of Seats				
Actual	0	33	118	8
Effect of $\Delta(\text{Income})$	[0.0, 0.8]	[-2.5, 8.4]	[-12.1, 2.0]	[-0.2, 1.2]
Effect of $\Delta(\text{Schooling})$	[-0.3, 0.5]	[0.0, 19.4]	[-19.2, 0.4]	[-1.5, 0.1]
Effect of $\Delta(\text{Age} > 65)$	[-0.6, 0.0]	[-13.2, 0.0]	[0.0, 13.2]	[-0.6, 0.6]

Table 5: Estimated Marginal Effects —  $\Delta(\text{Income})$  denotes changes of a 10% increase in income,  $\Delta(\text{Schooling})$  for a 2-year increase in years of schooling, and  $\Delta(\text{Age} > 65)$  for a 10% increase in the number of voters above age 65. We computed these numbers as in footnote 50.

are close in ideological space relative to the position of the LDP and the YUS, but compared with the JCP, the position of the DPJ is slightly closer to the LDP and the YUS. This is consistent with the general understanding that on the left-right spectrum, the JCP is very liberal, the DPJ is moderately liberal, and the LDP and the YUS are moderately conservative. Regarding voter positions, voters with lower income, fewer years of schooling, and older than 65 are ideologically closer to candidates from the LDP and the YUS than to candidates from the DPJ and the JCP. Regarding the second term of the ideological component (i.e., economic ideology), voters with lower income, longer years of schooling, and younger than 65 tend to be ideologically closer to pro-market candidates.

Second, the parameter estimates regarding candidate characteristics generally show the expected pattern. The estimates imply that incumbents are stronger than non-incumbents.<sup>48</sup> Also, the candidates enjoy a hometown advantage: the closer the hometown of the candidate is to the voter, the higher the voters utility in general.<sup>49</sup>

To interpret the magnitude of the estimated preference parameters, in Table 5 we present

<sup>48</sup> $\theta^{\text{incumbent}}$  measures the effect of being an incumbent,  $\theta^{\text{previous}}$  measures the effect of previously having held public office, and  $\theta^{\text{no-experience}}$  measures the effect of not having had any experience in public office, where  $\theta^{\text{no-experience}} = 0$ , by normalization.

<sup>49</sup>The parameter  $\theta^{\text{hometown1}}$  captures the effect of having a hometown in the same municipality as the voter, and  $\theta^{\text{hometown2}}$  is the effect of having a hometown in the same electoral district but in a different municipality.  $\theta^{\text{hometown3}}$  is the effect of having a hometown in the same prefecture as the voter but not in the same electoral district, and lastly,  $\theta^{\text{hometown4}} = 0$  is the effect of having a hometown in a different prefecture.

Predicted Closeness ( $w$ )	Average Fraction of Strategic Voters
1.0	[82.42%, 84.94%]
1.5	[79.36%, 83.04%]
2.0	[76.25%, 81.06%]
2.5	[73.11%, 79.02%]
3.0	[69.92%, 76.90%]
3.5	[66.68%, 74.70%]
4.0	[63.40%, 72.42%]

Table 6: Average Fraction of Strategic Voters by Predicted Closeness of the Election.

how the outcome would change if we move the exogenous demographics of the electorate. Specifically, we consider how the vote share and the number of seats are affected by the following three changes; 10% increase in the income level, two-year increase in years of schooling, and 10% increase in population of age above 65.<sup>50</sup> We find that the 10% increase in population of age above 65 increases the average vote share and number of seats for the LDP by between 0.03% and 0.95% and by between 0 and 13.2 seats, while it has the opposite effect on the DPJ with the decrease between -0.85% and -0.03% and between -13.2 and 0 seats. Two-year increase in education increases the average vote share and the number of seats for the DPJ by between 0.09% and 2.18% and by between 0 and 19.4 seats, while it decreases the vote share for the LDP by between -2.19% and -0.01%.

Finally, recall that we specified the distribution of  $\alpha$  to be a Beta distribution with parameters  $\theta_{\alpha 1} = \theta_{\alpha 1}^{const} + \theta_{\alpha 1}^{closeness} w$  and  $\theta_{\alpha 2} = \theta_{\alpha 2}^{const} + \theta_{\alpha 2}^{closeness} w$ , where  $w$  is an indicator of predicted closeness of the election taking values between 1 and 4. In Table 6, we report what these parameter values translate to in terms of the average fraction of strategic voters as a function of  $w$ . The fraction ranges from between 82.42% and 84.94% for the closest races to between 63.40% and 72.42% for the least competitive, which seems to indicate that the predicted closeness of the election and the fraction of strategic voters are positively related.

The average fraction of strategic voters that we report may seem surprisingly high given the fact that the fraction of strategic voting reported in previous studies is between 3% and 17%. However, note that the fraction of “strategic voting” reported in previous studies is in fact the fraction of misaligned voting, as discussed in the Introduction, and not the standard definition of strategic voting (See, e.g., the entry of “strategic voting” in *The New*

<sup>50</sup>To obtain the numbers in Table 5, we compute the change in the vote share and the number of seats for a given belief  $T^d$ , for each district by drawing the error terms  $(\alpha, \xi)$  from the estimated distribution. We then compute the min and max of these changes with respect to  $T^d$  in each district, and average them across districts (for vote share) or sum them across districts (for number of seats). The effect on the number of seats reported in Table 5 are not integers because we draw the error terms  $(\alpha, \xi)$  and take average over them.

*Palgrave Dictionary of Economics* by Feddersen (2008).). Misaligned voting is an equilibrium behavior of strategic voters, and strategic voters may or may not vote for their most preferred candidate. In order to compare our result with the previous studies, we use the estimated model to compute the extent of misaligned voting in the next subsection.

## 5.2 Extent of *Misaligned Voting*

The extent of misaligned voting is given by the fraction of voters who do not vote for the most preferred candidate. Because we do not have any individual voter-level voting records (we only observe vote shares at the municipality level), we still face the task of identifying the extent of misaligned voting from aggregate data. Identifying the extent of misaligned voting from aggregate data alone is not straightforward because there could be misaligned voting at the individual level, but the inflow of misaligned votes to candidate  $k$  (i.e., votes cast for candidate  $k$  by voters who do not prefer  $k$  the most) and the outflow of misaligned votes from candidate  $k$  (i.e., votes cast for a candidate other than  $k$  by voters who prefer  $k$  the most) may cancel each other out at the aggregate municipality level.

More precisely, let  $v_k^{data}$  denote the actual vote share for candidate  $k$  and let  $v_k^{sin}$  denote the predicted vote share for candidate  $k$  when everyone votes sincerely. Also, let  $D_{kl}$  denote the total votes cast for candidate  $k$  by strategic voters who prefer candidate  $l$  most (inflow of misaligned votes from  $l$  to  $k$ ). Then the object of interest, the amount of misaligned voting, can be expressed as  $\sum_{k,l} D_{kl}$ . On the other hand, because we only have aggregate data, the best we can do is to compute the net inflow/outflow,  $v_k^{data} - v_k^{sin}$ , instead of  $\sum_{k,l} D_{kl}$ . (Note that  $v_k^{data} - v_k^{sin} = \sum_l D_{kl} - \sum_l D_{lk}$  and  $\sum_l D_{kl}$  is the inflow of misaligned votes into candidate  $k$  and  $\sum_l D_{lk}$  is the outflow of misaligned votes from candidate  $k$ .) We show in Appendix C that we can still obtain bounds for  $\sum_{k,l} D_{kl}$  using  $v_k^{data} - v_k^{sin}$ .

In addition, computing  $v_k^{sin}$  alone involves some complexities. This is because (1) the realization of municipality level shocks ( $\xi$ ) cannot be uniquely recovered and (2) the model parameters are set identified. We describe how to deal with these issues in Appendix C.

We obtained the lower and upper bounds of misaligned voting as 1.15% and 2.67%. Given that we have estimated the average fraction of strategic voters to be between 63.40% and 72.42% in the least competitive races and between 82.42% and 84.94% in the closest races, this implies that between 1.37% and 4.21% of strategic voters engaged in misaligned voting. Our estimates of misaligned voting are roughly comparable to the numbers reported in the existing literature, ranging from 3% to 17%.<sup>51</sup>

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<sup>51</sup>The finding in existing studies are somewhat larger perhaps because the previous literature has focused mostly on settings that are particularly conducive to misaligned voting (primarily Canadian and British general elections). In any event, there is no a priori reason why the fraction of misaligned voting should be

	JCP	DPJ	LDP	YUS
Actual				
Vote Share (%)	7.6	38.6	49.7	34.9
Number of Seats	0	33	118	8
Counterfactual				
Vote Share (%)	[9.08, 14.02]	[33.35, 39.98]	[42.72, 51.11]	[35.36, 49.28]
Number of Seats	[1, 9]	[43, 61]	[79, 101]	[8, 15]

Table 7: Counterfactual Experiment – Sincere Voting under Plurality Rule. Actual vote share is computed by taking the average of the vote shares only over districts in which the party fielded a candidate. Thus, they do *not* add up to 100%.

Finally, before moving on to the counterfactual experiment, we provide the fit of the model. We simulated the vote shares and the winner of each district using the estimated model in a similar way as we have computed the predicted vote shares in step 3 of our estimation procedure.<sup>52</sup> We find that the predicted set of vote shares contains the actual vote shares in 83.6% of the districts, and the predicted set of winners contains the actual winner in 98.7% of the districts.

## 5.3 Counterfactual Experiment

### 5.3.1 Sincere Voting under Plurality Rule

In our counterfactual experiment, we investigate what the outcome would have been if all voters had voted sincerely under plurality rule. It is well known from Gibbard (1973) and Satterthwaite (1975) that there does not exist a strategy-proof voting mechanism (except for a dictatorial mechanism or a mechanism in which a particular candidate is never chosen under any circumstances). Even though a strategy-proof voting mechanism does not exist, we can simulate the hypothetical sincere-voting outcome under plurality rule using the estimated model. While we do not account for endogenous entry decisions of the parties that may entail,<sup>53</sup> this exercise still enables us to get a sense of the important role strategic voters play in determining the outcome in plurality rule elections.

Table 7 compares the actual vote shares and the number of seats with those of the sincere-

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similar across different elections, since misaligned voting is an equilibrium object.

<sup>52</sup>More precisely, we compute  $\min_{T^d \in \mathfrak{T}(v_d^{data})} v_d^{PRED}(T^d)$  and  $\max_{T^d \in \mathfrak{T}(v_d^{data})} v_d^{PRED}(T^d)$  for each electoral district, then calculate the 2.5-percentile and the 97.5-percentile (with respect to realizations of  $(\alpha, \xi)$ ).

<sup>53</sup>If we were to account for endogenous entry on the part of the parties, the gain in the vote share for DPJ may be smaller and the loss for the LDP may be even bigger. This is because more parties may enter and take votes away from the existing parties.

voting experiment (Note that the vote shares do not add up to 100% because the vote shares are computed by taking the average of the vote shares only over districts in which the party fielded a candidate). The details on how we obtained Table 7 are provided in Appendix D.

We find that the number of seats for the DPJ and the LDP change significantly in spite of the fact that the extent of misaligned voting is relatively small. The DPJ would add between 10 and 28 seats and the LDP would lose between 17 and 39 seats. Compared to the relatively small change in the vote share, the change in the number of seats is considerable. Note that this difference in the number of seats is accounted for by misaligned voting. Even though the extent of misaligned voting is small, the impact on the number of seats is large because the winning margin is often small.

With respect to vote shares, we find that the vote shares for the JCP and the YUS generally increase in our experiment, implying that the two main parties, the DPJ and the LDP probably benefited from misaligned voters of the JCP and the YUS in the actual election. Even though both DPJ and LDP might have gained from misaligned voters in terms of vote share, it seems that the LDP benefited a lot more than the DPJ in terms of the number of seats: While neither of the two parties seem to lose much vote share in the counterfactual, the LDP would lose between 17 and 39 seats in the counterfactual whereas the DPJ would gain between 10 and 28 seats.

## 6 Concluding Remarks

In this paper, we study how to identify and estimate a model of strategic voting and quantify its impact on election outcomes by adopting an inequality-based estimator. Preference and voting behavior do not necessarily have a one-to-one correspondence for strategic voters, and we obtain partial identification of preference parameters from the restriction that voting for the least preferred candidate is a weakly dominated strategy. The extent of strategic voting is identified using particular features of general-election data. We also make a clear distinction between strategic voting and misaligned voting.

By using aggregate data from the Japanese general election, we find that a large proportion of voters are strategic voters. We estimate the fraction of strategic voters to be between 63.4% and 84.9%, on average. In our counterfactual experiment, which assumes sincere voting by all voters under plurality, we find that the number of seats for the parties change significantly. Even though the extent of misaligned voting is small between 1.4% and 4.2%, the impact on the number of seats is considerable because the winning margin is often small.

One of the important issues that we did not deal with in this paper is voter turnout.

Voters' beliefs on pivot events are also important for models of voter turnout, and it may be possible to extend our approach in this direction. We leave this for future research.

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## 7 Appendix

### 7.1 Appendix A: Existence of Solution Outcome

We provide a proof of the existence of the solution outcome. It is almost identical to the proof in MW. Take some  $\varepsilon \in (0, 1)$ . We define a mapping  $\Phi$  from the product space of vote shares  $\mathcal{V}(= \Delta^{K \times M})$  and tie probability  $\mathcal{T}(= \Delta^{C_2})$  to its power set  $2^{\mathcal{V} \times \mathcal{T}}$  so that the fixed point of the mapping is an element in the solution outcome. Before we define  $\Phi$ , let us first define  $\Phi_1$  to be a mapping from  $\mathcal{V} \ni V = (V_1, \dots, V_K)$  to  $2^{\mathcal{T}}$ :  $\Phi_1(V) = \{T \in \mathcal{T} | V_k > V_l \Rightarrow T_{kn} \geq \varepsilon T_{ln} \forall k, l, n\}$ .  $\Phi_1$  is the set of tie probability that satisfy a stronger version of **C1** (because  $\varepsilon \in (0, 1)$ ).  $\Phi_1$  is non-empty valued, convex-valued and upper-hemi continuous. Now define  $\Phi_2$  to be a mapping from  $\mathcal{T}$  to  $2^{\mathcal{V}}$  as  $\Phi_2(T) = \{((V_{k,m}(T))_{k=1}^K)_{m=1}^M\}$  where  $V_{k,m}(T)$  is defined by **C2**.  $\Phi_2(T)$  is a singleton set.  $\Phi_2$  is also non-empty valued, convex valued and upper-hemi continuous. Now we define  $\Phi : \mathcal{V} \times \mathcal{T} \ni (V, T) \mapsto \Phi(V, T) = (\Phi_2(T), \Phi_1(V)) \in 2^{\mathcal{V} \times \mathcal{T}}$ . Then  $\Phi$  is also non-empty, convex-valued, and upper-hemi continuous. By applying Kakutani’s fixed point theorem to  $\Phi$ , we know that there exists a fixed point of  $\Phi$ . As the fixed point satisfies **C1** and **C2**, the solution outcome is nonempty.

### 7.2 Appendix B: Estimation

We use municipality-level aggregate data for our estimation. We denote the vote-share data of candidate  $k$  in municipality  $m$  by  $v_{k,m}^{data}$ . We use  $f_m$  to denote the distribution of demographic characteristics  $\mathbf{x}$  in municipality  $m$ . We let  $\boldsymbol{\varepsilon}_n = (\varepsilon_{nk})_{k=1}^K$  denote the  $K$  draws of individual-candidate-specific shock, and we let  $g$  denote the distribution of  $\boldsymbol{\varepsilon}_n$ . Similarly, denote  $\boldsymbol{\xi}_m = (\xi_{km})_{k=1}^K$ . Lastly, candidate  $k$ ’s characteristics are denoted by  $\mathbf{z}_{km}$ .

Recall that as in equation (3) we can express the vote share for candidate  $k$  in municipality  $m$  as a composition of vote shares among strategic and sincere voters:

$$v_{k,m}^{data} \approx (1 - \alpha_m)v_{k,m}^{SIN}(\boldsymbol{\xi}_m; \theta_0) + \alpha_mv_{k,m}^{STR}(T^d, \boldsymbol{\xi}_m; \theta_0) \quad (4)$$

where

$$\begin{aligned} v_{k,m}^{SIN}(\xi_m; \theta_0) &= \iint \mathbf{1}\{u_{nk} \geq u_{nl}, \forall l\} g(\varepsilon) d\varepsilon f_m(\mathbf{x}) d\mathbf{x} \\ v_{k,m}^{STR}(T^d, \xi_m; \theta_0) &= \iint \mathbf{1}\{\bar{u}_{nk}(T^d) \geq \bar{u}_{nl}(T^d), \forall l\} g(\varepsilon) d\varepsilon f_m(\mathbf{x}) d\mathbf{x}, \end{aligned}$$

are the expression for the vote share for candidate  $k$  among sincere and strategic voters.

Now, we construct moment inequalities based on the regression coefficients in each electoral district.

**Step 1** Take some  $\mathbf{z}$  and some district  $d$ . We obtain  $\beta_{k,d}^{data}$  by regressing the vote share data  $(v_{k,1}^{data}, \dots, v_{k,M^d}^{data})$  on the demographics in each municipality  $(f_1, \dots, f_{M^d})$ ,<sup>54</sup> i.e.,

$$\beta_{k,d}^{data} = \arg \min_{\beta} \left[ \sum_{m=1}^{M^d} \mathbf{1}_{\{z_{km}=\mathbf{z}\}} (v_{km}^{data} - \beta \cdot f_m)^2 \right].$$

Because  $M^d$  is not large, we cannot include many regressors. The number of regressors must be less than  $M^d$ . For this reason, we run 9 different types of regressions all involving just a constant or a constant and one component of  $f_m$ . For example, we run a regression of  $v_{km}^{data}$  on a constant and the fraction of population above 65 years old conditioned on  $z_{km} = LDP$ . The full set of regressions we use is in the Supplementary Material.

**Step 2** Fix some parameter  $\theta$ , beliefs  $T^d$ , and values of  $\alpha_d = \{\alpha_m\}_{m=1}^{M^d}$  and  $\xi_d = \{\xi_m\}_{m=1}^{M^d}$ . We can compute the vote shares for candidate  $k$  in each of the municipalities which we denote as  $(v_{k,1}^{PRED}(T^d, \alpha_1, \xi_1; \theta), \dots, v_{k,M^d}^{PRED}(T^d, \alpha_{M^d}, \xi_{M^d}; \theta))$ . We can obtain a closed form solution for the predicted vote share of sincere voters because  $\varepsilon$  is distributed type 1 extreme value. Regarding strategic voters, the predicted vote share does not have a closed form solution, and we use Monte-Carlo integration. For Monte-Carlo integration, we take 10 draws of  $\varepsilon$  for each demographic characteristics,  $\mathbf{x}$ . As we group the voters into 32 types according to their characteristics  $\mathbf{x}$ ,<sup>55</sup> we take 320 draws of  $\varepsilon$  for each municipality.

<sup>54</sup>As in Footnote 43, we can identify the distribution of demographic characteristics  $f_m$  with a vector of probabilities. We use the same notation  $f_m$  to denote the distribution and the vector of probabilities. The vector  $f_m$  contains, for example, the fraction of the population above 65, the fraction of population in different income ranges, etc.

<sup>55</sup>We discretize income into four groups, age into two groups, and education into four groups. Thus, we have  $4 \times 2 \times 4 = 32$  types.

**Step 3** Parallel to Step 1, regress the simulated vote shares of candidate  $k$ ,  $(v_{k,1}^{PRED}(T^d, \alpha_1, \boldsymbol{\xi}_1; \theta), \dots, v_{k,M^d}^{PRED}(T^d, \alpha_{M^d}, \boldsymbol{\xi}_{M^d}; \theta))$ , on the demographic characteristics in each municipality  $(f_1, \dots, f_{M^d})$ , conditioning on a particular value of  $z$ . We obtain the regression coefficient as

$$\beta_{k,d}(T^d, \boldsymbol{\alpha}_d, \boldsymbol{\xi}_d; \theta) = \arg \min_{\beta} \left[ \sum_{m=1}^{M^d} 1_{\{z_{km}=z\}} (v_{k,m}^{PRED}(T^d, \alpha_{M^d}, \boldsymbol{\xi}_{M^d}; \theta) - \beta \cdot f_m)^2 \right].$$

**Step 4** Because we do not know  $T^d$ , we vary  $T^d \in \mathfrak{T}(v^{data})$  to obtain the minimum and maximum values of the regression coefficients  $\underline{\beta}_{k,d}(\boldsymbol{\alpha}_d, \boldsymbol{\xi}_d; \theta)$  and  $\overline{\beta}_{k,d}(\boldsymbol{\alpha}_d, \boldsymbol{\xi}_d; \theta)$  as in the main text. In practice, we discretize  $\mathfrak{T}(v^{data})$  with a grid size equal to 0.04.

**Step 5** We integrate out  $\boldsymbol{\alpha}_d$  and  $\boldsymbol{\xi}_d$  by simulating values of  $\boldsymbol{\alpha}_d$  and  $\boldsymbol{\xi}_d$  from  $F_{\alpha}$  and  $F_{\xi}$ , and obtain  $\overline{\beta}_{k,d}(\theta)$  and  $\underline{\beta}_{k,d}(\theta)$ , as defined in the main text. We draw 10 realizations of  $\alpha_m$  and  $\xi_m$  from  $F_{\alpha}$  and  $F_{\xi}$ , hence we have  $10 \times M^d$  draws for each district  $d$ .

**Step 6** We take the average of  $\overline{\beta}_{k,d}(\theta)$ ,  $\underline{\beta}_{k,d}(\theta)$  and  $\beta_{k,d}^{data}$  across  $d$  and obtain the empirical analog as in the main text.

Finally, to improve the sharpness of the identified set, we include another type of moment inequalities that harnesses the comovements in  $\beta$  that results from varying  $T$ . Notice that in Step 4, we have computed the maximum and the minimum values of  $\beta$  separately for each of the 9 types of regressions. But note that the coefficients from the regressions cannot move independently. Thus in an effort to use some of these restrictions, we can construct additional moment inequalities by taking linear combination of  $\beta$ . For example, let  $\beta_{k,d}^{OLD}$  and  $\beta_{k,d}^{RICH}$  be the regression coefficients that we obtain in Steps 1 and 4 when we regress vote shares on the proportion of the population above 65 and the proportion of the population in the highest income quartile, respectively. Then we can consider  $\max_{\{T^d\}} (\beta_{k,d}^{OLD}(T^d) - \beta_{k,d}^{RICH}(T^d))$  and use this to form moment inequalities. More generally, for any matrix  $A$ , we can consider  $\overline{A\beta}_{k,d} \equiv \max_{\{T^d\}} A\beta_{k,d}(T^d)$  and  $\underline{A\beta}_{k,d} \equiv \min_{\{T\}} A\beta_{k,d}(T^d)$  and construct moment inequalities by following the same argument presented in the main text. We provide the exact form of matrix  $A$  that we use in our estimation in the Supplementary Material.

### 7.3 Appendix C: Computation of Misaligned Voting

The amount of misaligned voting is given by the fraction of voters who do not vote for the most preferred candidate. As we discussed in the main text, we do not have any individual voting records (we only observe vote shares at the municipality level), so we need to identify

the extent of misaligned voting from aggregate data. In Step 1, we discuss issues arising from identifying the extent of misaligned voting from aggregated data, assuming that we can precisely recover the outcome when everyone votes sincerely. Then, in Steps 2 to 4, we will discuss issues related to recovering the sincere voting outcome from the estimated model.

### Step 1

Let  $v_k^{data}$  denote the actual vote share for candidate  $k$  and let  $v_k^{sin}$  denote the vote share of candidate  $k$  when everyone votes sincerely (subscripts  $d, m$  are suppressed from now on). Also, let  $D_{kl}$  denote the total votes cast for candidate  $k$  by strategic voters who prefer candidate  $l$  most (inflow/outflow of misaligned votes from  $l$  to  $k$ ). Then the object of interest, the amount of misaligned voting, can be expressed as  $\sum_{k,l} D_{kl}$ . On the other hand, the available information is summarized as  $v_k^{data} - v_k^{sin} = \sum_l D_{kl} - \sum_l D_{lk}$ , where  $\sum_l D_{kl}$  is the inflow of misaligned votes into candidate  $k$  and  $\sum_l D_{lk}$  is the outflow of misaligned votes from candidate  $k$ . (Note that **C1** implies that if  $D_{kl} > 0$ , then  $D_{lk} = 0$ ). The question we are concerned with is the following: What can we learn about  $\sum_{k,l} D_{kl}$  given that we only know  $v_k^{data} - v_k^{sin} (\equiv \Delta^k) = \sum_l D_{kl} - \sum_l D_{lk}$ ?

We can show that for  $K = 3$ ,  $\sum_{k,l} D_{kl}$  can be bounded below by

$$lb(\{\Delta^k\}) = \max_k \{|\Delta^k|\}$$

and above by

$$ub(\{\Delta^k\}) = \max_k \{\Delta^k\} - \min_k \{\Delta^k\}.$$

We provide an analogous expression for  $K = 4$  in the Supplementary material. These bounds are also sharp among all bounds that can be obtained without imposing any distributional assumptions on the shocks in the utility function.<sup>56</sup> The proofs are provided in the Supplementary material.

### Step 2 to Step 4

Now we discuss issues related to recovering the sincere voting outcome from the estimated model. Given preference parameters of the model, for any realization of  $\xi$ , we can compute what the outcome would be if all voters vote sincerely. We denote this predicted sincere-voting outcome as  $v^{sin}(\hat{\theta}, \xi)$ . Ideally, we would know the actual realization of  $\xi$ ,  $\xi = \xi_0$  in each municipality, and compute the sincere voting outcome,  $v^{sin}(\hat{\theta}, \xi_0)$ , using this actual

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<sup>56</sup>We do not know whether the bounds are sharp with regard to the class of DGPs that we considered in our estimation where we have imposed distributional assumptions on the unobservable shocks in the utility function. As our estimation bypasses inference on  $T$ , it is difficult to obtain bounds that are, at the same time, computable and sharp with regard to the DGPs we considered in the estimation.

realization of  $\xi_0$  and using a parameter value in the estimated set,  $\hat{\theta} \in \hat{\Theta}_{CI}$ . Then the difference between the observed vote share,  $v^{data}$  and  $v^{sin}(\hat{\theta}, \xi_0)$ , ( $\Delta^k = v^{data} - v^{sin}(\hat{\theta}, \xi_0)$ ) would allow us to compute the lower and upper bounds,  $lb(\{\Delta^k\})$  and  $ub(\{\Delta^k\})$ . However,  $\xi_0$  cannot be recovered uniquely. Also, the difference between  $v^{data} = v(\xi_0)$  and  $v^{sin}(\hat{\theta}, \xi)$  depends on  $\hat{\theta}$ , which we have only set-identified. Hence, we compute the bounds on the extent of misaligned voting in the following three steps (Step 2 to Step 4).

In Step 2, fix  $\hat{\theta} \in \hat{\Theta}_{CI}$ . For any given draw of  $\xi$  from  $\hat{F}_\xi$ , we compute  $\hat{\Delta}^k(\xi)$ ,

$$\hat{\Delta}^k(\xi) = v_k^{data} - v_k^{sin}(\hat{\theta}, \xi)$$

and the corresponding bounds  $lb(\{\hat{\Delta}^k(\xi)\})$  and  $ub(\{\hat{\Delta}^k(\xi)\})$ . By Monte Carlo, we then compute the expected value of the bounds where the expectation is taken with regard to the randomness in  $\xi$ ,

$$\begin{aligned} Lb_0 &= \int lb(\{\hat{\Delta}^k(\xi)\}) d\hat{F}_\xi(\xi), \text{ and} \\ Ub_0 &= \int ub(\{\hat{\Delta}^k(\xi)\}) d\hat{F}_\xi(\xi), \end{aligned}$$

for each municipality, where  $\hat{F}_\xi$  is the estimated distribution of  $\xi$ . Note that  $Lb_0$  and  $Ub_0$  do not necessarily coincide with  $lb(\{\hat{\Delta}^k(\xi_0)\})$  and  $ub(\{\hat{\Delta}^k(\xi_0)\})$ , which are the lower and upper bounds of the extent of misaligned voting we would obtain if we had full knowledge of the realizations of  $\xi$ ,  $\xi = \xi_0$ . Therefore, we need to account for the parts of  $Lb_0$  and  $Ub_0$  that are induced by the randomness in  $\xi$ . We discuss this in Step 3.

In Step 3, we evaluate the components of  $Lb_0$  and  $Ub_0$  that are induced by the randomness in  $\xi$ . To do so, we compute the mean effects of the randomness components by calculating (using Monte Carlo integration)

$$\begin{aligned} Lb_\xi &= \int \int lb(\{\tilde{\Delta}^k(\tilde{\xi}, \tilde{\xi})\}) d\hat{F}_\xi(\tilde{\xi}) d\hat{F}_\xi(\tilde{\xi}), \text{ and} \\ Ub_\xi &= \int \int ub(\{\tilde{\Delta}^k(\tilde{\xi}, \tilde{\xi})\}) d\hat{F}_\xi(\tilde{\xi}) d\hat{F}_\xi(\tilde{\xi}), \end{aligned}$$

where  $\tilde{\Delta}^k(\tilde{\xi}, \tilde{\xi})$  is the difference in the vote share between two realizations of municipality-level shock,  $\tilde{\xi}$  and  $\tilde{\xi}$ , i.e.,

$$\tilde{\Delta}^k(\tilde{\xi}, \tilde{\xi}) = v_k^{sin}(\hat{\theta}, \tilde{\xi}) - v_k^{sin}(\hat{\theta}, \tilde{\xi}).$$

We then compute the lower and upper bounds of misaligned voting at the municipality level

as

$$\begin{aligned} LB &= Lb_0 - Lb_{\xi}, \text{ and} \\ UB &= Ub_0 - Ub_{\xi}. \end{aligned}$$

In Step 4, we account for the fact that  $\theta$  is only set-identified. So far, we have been computing  $LB$  and  $UB$  implicitly treating  $\theta$  as given. By denoting the dependence on  $\theta$  more explicitly,  $LB$  and  $UB$  above can be written as  $LB(\theta)$  and  $UB(\theta)$ . Because  $\theta$  is partially identified, we need to compute  $LB(\theta)$  and  $UB(\theta)$  by allowing  $\theta$  to move in the partially identified set  $\Theta_{CI}$  in order to construct the most conservative bound on the extent of misaligned voting,  $\underline{LB}$  and  $\overline{UB}$ , i.e.

$$\begin{aligned} \underline{LB} &= \min_{\theta \in \Theta_{CI}} LB(\theta), \text{ and} \\ \overline{UB} &= \max_{\theta \in \Theta_{CI}} UB(\theta). \end{aligned}$$

## 7.4 Appendix D: Computation of Second Counterfactual

Computation of the second counterfactual proceeds in the same way as described in Steps 2 to 4 in Appendix B. This is because as was the case in our first counterfactual, we cannot recover the realization of the municipality level random shock  $\xi, \xi = \xi_0$ . Denote the counterfactual vote share as  $v^{sin}(\hat{\theta}, \xi_0)$ . The problem is that we cannot compute this because  $\xi_0$  is unobserved. But we can obtain bounds for  $v^{sin}(\hat{\theta}, \xi_0)$  by following the same procedure as in Appendix C. We can also compute the number of seats in the same way. Note that we do not need to take Step 1 in this case.

## 7.5 Supplementary Material A

In our estimation, we run regressions in Step 1 and Step 3 in order to obtain  $\beta_{k,d}^{data}$  and  $\beta_{k,d}(T^d, \alpha_d, \xi_d; \theta)$ , which are

$$\begin{aligned}\beta_{k,d}^{data}(T^d, \alpha_d, \xi_d; \theta) &= \arg \min_{\beta} \left[ \sum_{m=1}^{M^d} 1_{\{z_{km}=z\}} (v_{k,m}^{data} - \beta \cdot f_m)^2 \right], \text{ and} \\ \beta_{k,d}(T^d, \alpha_d, \xi_d; \theta) &= \arg \min_{\beta} \left[ \sum_{m=1}^{M^d} 1_{\{z_{km}=z\}} (v_{k,m}^{PRED}(T^d, \alpha_{M^d}, \xi_{M^d}; \theta) - \beta \cdot f_m)^2 \right].\end{aligned}$$

We run 9 different types of regressions (fourty eight regressions in total) for each district as follows.

1.  $f_m = (1, \text{"fraction of population above 65"})$ , i.e. we regress the vote shares onto a constant and the fraction of population above 65 years old. If we let  $P$  denote  $\{LDP, DPJ, JCP, YUS\}$ , we run this regression for each combination of  $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$ .
2.  $f_m$  is a constant and the fraction of population with years of schooling between 12 to 14 years. Regression is run for each combination of  $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$ .
3.  $f_m$  is a constant and the fraction of population with years of schooling between 15 to 16 years. Regression is run for each combination of  $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$ .
4.  $f_m$  is a constant and the fraction of population with years of schooling over 16 years. Regression is run for each combination of  $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$ .
5.  $f_m$  is a constant and the fraction of population with income in the first quartile (lower than 1.870 million yen). Regression is run for each combination of  $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$ .
6.  $f_m$  is a constant and the fraction of population with income in the second quartile (between 1.870 million yen and 2.704 million yen). Regression is run for each combination of  $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$ .
7.  $f_m$  is a constant and the fraction of population with income in the third quartile (between 2.704 million yen and 3.911 million yen). Regression is run for each combination of  $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$ .
8.  $f_m$  is a constant. Regression is run for each combination of  $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$ .
9.  $f_m$  is a constant. Regression is run for each combination of  $(z_1^{POS}, \dots, z_K^{POS}) \in P^K$ , and  $(z_{km}^{EXPR}, z_{km}^{HOME})$  where  $z_{km}^{EXPR} \in \{\text{incumbent, previous political experience, no previous political experience}\}$ , and  $z_{km}^{HOME} \in \{\text{hometown of the candidate is outside the prefecture, hometown of the candidate is inside the prefecture (but outside the district), hometown of the candidate is in the district (but outside municipality } m), \text{hometown of the candidate is in municipality } m\}$ .

In order to improve the sharpness of the identified set, we include another type of moment inequalities that harnesses the comovements in  $\beta$  that results from changes in  $T$  as discussed in Step 6 of Appendix B. We augment the moment conditions by using restrictions on the comovement of the coefficients for the 9th type of regressions. This allows us to add restrictions on the pairwise difference in the  $\beta$ s that relate to the effect of candidates' experience and hometowns, e.g., the difference in the vote share for a *LDP* candidate whose hometown is outside of the prefecture compared to a *LDP* candidate whose hometown is within the prefecture. In practice, the matrix  $A$  used in Step 6 in our estimation is  $A^T = \begin{pmatrix} I_{60} & \mathbf{0} \\ & B \end{pmatrix}$

where  $B = \begin{pmatrix} 1 & \cdots & \cdots & 1 & 0 & \cdots & 0 & \cdots & \cdots \\ -1 & 0 & \cdots & 0 & 1 & \cdots & 1 & 0 & \cdots \\ 0 & -1 & \ddots & \vdots & -1 & 0 & 0 & 1 & \cdots \\ \vdots & \ddots & \ddots & 0 & 0 & \ddots & 0 & -1 & 0 \\ 0 & \cdots & 0 & -1 & \vdots & \ddots & -1 & 0 & \ddots \end{pmatrix}$  and  $I_{60}$  is the identity matrix of size  $60 \times 60$ .

## 8 Supplementary Material B

In this Supplementary Material, we prove that the bounds  $ub(\{\Delta^k\})$  and  $lb(\{\Delta^k\})$  we have used in Appendix C in fact constitute bounds and that they are sharp. Because the bounds are different for  $K = 3$  and  $K = 4$ , we prove each case in turn. We drop subscripts  $d$  and  $m$  for the rest of the section.

### 8.0.1 Case of $K = 3$

First, we prove that, for the case of  $K = 3$ , the extent of strategic voting is bound by  $lb(\{\Delta^k\})$  and  $ub(\{\Delta^k\})$ , where

$$\begin{aligned} lb(\{\Delta^k\}) &= \max_k \{|\Delta^k|\}, \text{ and} \\ ub(\{\Delta^k\}) &= 1\{\#\{\Delta^k > 0\} = 2\}(\max_k \{\Delta^k | \Delta^k > 0\} - \min_k \{\Delta^k\}) \\ &\quad + 1\{\#\{\Delta^k > 0\} = 1\}(\max_k \{\Delta^k\} - \min_k \{\Delta^k | \Delta^k < 0\}) \\ &= \max_k \{\Delta^k\} - \min_k \{\Delta^k\}, \end{aligned}$$

and  $\#\{\Delta^k > 0\}$  indicates the number of  $\Delta^k$ s that are positive, and  $1\{\cdot\}$  is an indicator function. Let  $D_{kl}$  denote the votes cast for candidate  $k$  by strategic voters who prefers candidate  $l$  most. Then the amount of misaligned voting is  $\sum_{kl} D_{kl}$  (Note that **C1** implies



that if  $D_{kl} > 0$ , then  $D_{lk} = 0$ ).

First, we prove that the extent of strategic voting is bound by  $lb(\{\Delta^k\})$  and  $ub(\{\Delta^k\})$ . Without loss of generality, index the candidates as 1, 2, and 3 such that the beliefs regarding the tie probabilities satisfy  $T_{12} \geq T_{13} \geq T_{23}$ . Then the amount of misaligned voting is  $D = D_{12} + D_{13} + D_{23}$  (Note that  $D_{21} = D_{31} = D_{32} = 0$ ). Now, we can write

$$\Delta^1 = D_{12} + D_{13}, \quad (\text{A1})$$

$$\Delta^2 = D_{23} - D_{12}, \quad (\text{A2})$$

$$\Delta^3 = -D_{13} - D_{23}. \quad (\text{A3})$$

Note that  $|\Delta^1| + |\Delta^3| = D_{12} + 2D_{13} + D_{23} \geq D$ , thus  $|\Delta^1| + |\Delta^3|$  is an upper bound. We consider two cases; (i)  $\#\{\Delta^k > 0\} = 1$ , and (ii)  $\#\{\Delta^k > 0\} = 2$ . In case (i), we know that the positive number we observe is  $\Delta^1$ , but cannot identify which of the two negative numbers correspond to  $\Delta^2$  or  $\Delta^3$ . In case (ii), we know that the negative number we observe is  $\Delta^3$ , but we cannot identify which of the two positive numbers correspond to  $\Delta^1$  or  $\Delta^2$ . These two cases are exhaustive as  $\Delta^1 + \Delta^2 + \Delta^3 = 0$ . In case (i),

$$\begin{aligned} ub(\{\Delta^k\}) &= \max_k \{\Delta^k\} - \min_k \{\Delta^k | \Delta^k < 0\} = \Delta^1 - \min\{\Delta^2, \Delta^3\} \\ &= |\Delta^1| + \max\{|\Delta^2|, |\Delta^3|\} \\ &\geq |\Delta^1| + |\Delta^3|. \end{aligned}$$

In case (ii),

$$\begin{aligned} ub(\{\Delta^k\}) &= \max_k \{\Delta^k | \Delta^k > 0\} - \min_k \{\Delta^k\} = \max\{\Delta^1, \Delta^2\} - \Delta^3 \\ &= \max\{|\Delta^1|, |\Delta^2|\} + |\Delta^3| \\ &\geq |\Delta^1| + |\Delta^3|. \end{aligned}$$

We can also see that  $\max_k \{|\Delta^k|\}$  is the lower bound because  $|\Delta^1| = D_{12} + D_{13} \leq D$ ,  $|\Delta^2| \leq D_{23} + D_{12} \leq D$ , and  $|\Delta^3| = D_{13} + D_{23} \leq D$ .

Second, we prove by contradiction that the upper bound  $ub(\{\Delta^k\})$  is sharp. Let  $h(\Delta^1, \Delta^2, \Delta^3) \leq ub(\{\Delta^k\})$  for all  $\{\Delta_{d,m}^k\}$ , and moreover  $h(\Delta^{1*}, \Delta^{2*}, \Delta^{3*}) < ub(\{\Delta^k\})$ . Without loss of generality, consider the following two cases (i)  $\Delta^{1*} > 0 > \max\{\Delta^{2*}, \Delta^{3*}\}$  and (ii)  $\min\{\Delta^{1*}, \Delta^{2*}\} > 0 > \Delta^{3*}$ . Note that we cannot identify whether the two negative numbers in case (i) correspond to  $\Delta^{2*}$  or  $\Delta^{3*}$ , and similarly, in case (ii), we cannot identify whether the two positive numbers correspond to  $\Delta^{1*}$  or  $\Delta^{2*}$ . This is the reason why we use the min and the max operators. In case (i), if we let  $D_{12} = \Delta^{1*}$ ,  $D_{23} = -\min\{\Delta^{2*}, \Delta^{3*}\}$  and  $D_{13} = 0$ , then the three

equations (A1)-(A3) can be satisfied. In this instance,  $D_{12} + D_{13} + D_{23} = \Delta^{1*} - \min\{\Delta^{2*}, \Delta^{3*}\} = ub(\{\Delta^{k*}\})$ , achieving our bound. Hence,  $h$  cannot be an upper bound. In case (ii), let  $D_{12} = \max\{\Delta^{1*}, \Delta^{2*}\}$ ,  $D_{13} = 0$ ,  $D_{23} = -\Delta^{3*}$ . Then (A1)-(A3) are satisfied, and moreover,  $D_{12} + D_{13} + D_{23} = \max\{\Delta^{1*}, \Delta^{2*}\} - \Delta^{3*} = ub(\{\Delta^{k*}\})$ .

Third, we prove by contradiction that the lower bound  $lb(\{\Delta^k\})$  is sharp. Let  $h(\Delta^1, \Delta^2, \Delta^3) \geq lb(\{\Delta^k\})$  for all  $\{\Delta_{d,m}^k\}$ , and moreover  $h(\Delta^{1*}, \Delta^{2*}, \Delta^{3*}) > lb(\{\Delta^k\})$ . Without loss of generality, consider the following two cases (i)  $\Delta^{1*} > 0 > \max\{\Delta^{2*}, \Delta^{3*}\}$  and (ii)  $\min\{\Delta^{1*}, \Delta^{2*}\} > 0 > \Delta^{3*}$ . In case (i), let  $D_{12} = -\Delta^{2*}$ ,  $D_{13} = -\Delta^{3*}$ , and  $D_{23} = 0$ . This satisfies the three equations (A1)-(A3) and moreover,  $D_{12} + D_{13} + D_{23} = -\Delta^{2*} - \Delta^{3*} = \Delta^{1*} = lb(\{\Delta^{k*}\})$ . In case (ii) let  $D_{12} = 0$  and  $D_{23} = \Delta^{2*}$  and  $D_{13} = -\Delta^{3*} - \Delta^{2*}$ . This also satisfies equations (A1)-(A3), and implies  $D_{12} + D_{13} + D_{23} = -\Delta^{3*} = lb(\{\Delta^{k*}\})$ . Thus,  $h$  cannot be a lower bound.

### 8.0.2 Case of $K = 4$

For the case of  $K = 4$ , the lower and upper bounds  $lb(\{\Delta^k\})$  and  $ub(\{\Delta^k\})$  are written as

$$\begin{aligned} lb(\{\Delta^k\}) &= 1\{\#\{\Delta^k > 0\} = 3\} \max \left\{ \min_{k,l \neq k} \{\Delta^k + \Delta^l | \Delta^k, \Delta^l > 0\}, -\min_k \{\Delta^k | \Delta^k < 0\} \right\} \\ &\quad + 1\{\#\{\Delta^k > 0\} = 2\} \max \left\{ \min_k \{\Delta^k | \Delta^k > 0\}, -\min_k \{\Delta^k | \Delta^k < 0\} \right\} \\ &\quad + 1\{\#\{\Delta^k > 0\} = 1\} \max \left\{ \max_k \{\Delta^k\}, -\max_k \{\Delta^k | \Delta^k < 0\} \right\}, \text{ and} \\ ub(\{\Delta^k\}) &= \max_{k,l \neq k} \{2\Delta^k + \Delta^l\} - \max_k \{\Delta^k | \Delta^k < 0\} \end{aligned}$$

The proof is similar to the case of  $K = 3$ .