Scoring and Cartel Discipline in Procurement Auctions*

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April 10, 2025

Abstract

Auctioneers suspecting bidder collusion often lack the formal evidence needed for legal recourse. A practical alternative is to design auctions that hinder collusion. Since Abreu et al. (1986), economic theory has emphasized imperfect monitoring as a constraint on collusion, but evidence remains scarce on whether: (i) information frictions meaningfully limit real-world collusion; and (ii) auctioneers can effectively exploit these frictions. Indeed, transparency concerns prevent the introduction of explicit randomness in auction design. We make progress on this issue by studying the impact of subjective scoring in auctions run by Japan's Ministry of Land, Infrastructure, and Transportation. The adoption of scoring auctions significantly reduced winning bids in ways inconsistent with competition. Model-based inference suggests that the cartel's dynamic obedience constraints were binding and tightened by imperfect monitoring. Subjective scoring can successfully leverage imperfect monitoring frictions to reduce the scope of collusion.

Keywords: procurement, scoring, cartel discipline, imperfect monitoring.

*We are grateful to audience members at Berkeley, Boston University, MIT Sloan, the University of Wisconsin, the 2023 Econometric Society's Theory Summer School, and the 2024 Cowles Conference on Models and Measurement for helpful feedback.

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1 Introduction

Procurement agencies suspecting collusion among bidders can potentially respond by introducing frictions known to make repeated game cooperation more difficult. This includes encouraging entry (Starc and Wollmann, 2022), or reducing the cartel's enforcement capacity through minimum price guarantees (Chassang and Ortner, 2019). The repeated games literature (Green and Porter, 1984, Abreu et al., 1986, 1990, Kandori, 1992, Fudenberg et al., 1994) has emphasized imperfect monitoring as a key friction limiting parties' ability to collude, but there is little practical evidence that procurement agencies can successfully exploit this type of friction to address suspected collusion. Good governance principles require public procurement agencies to use clear and transparent processes as well as maintain public bidding records, which facilitates monitoring among cartel members. This paper identifies subjective scoring as an implicit source of imperfect monitoring in procurement auctions, and argues empirically that it can place additional binding constraints on cartel discipline in a real-life procurement context. This identifies an additional motive for the use of scoring auctions, and suggests that limits to transparency may sometimes be beneficial.

Our analysis exploits a change in the procurement mechanism used by Japan's Ministry of Land, Infrastructure and Transportation (MLIT). Prior to 2005, Japan's MLIT used price-only auctions to allocate contracts. Starting in 2005, the MLIT gradually shifted towards scoring auctions. Strikingly, winning bids decreased significantly following the introduction of scoring auctions, which goes against competitive bidding predictions: scoring should increase both quality and prices. This motivates us to study the impact of scoring on collusive auctions.

We interpret auction outcomes through a repeated game model. A group of firms repeatedly participates in procurement auctions. Firms' types are drawn i.i.d. across periods, and are publicly observed among firms. At the end of each period, after the outcome of the auction is realized, firms are able to make costly transfers to each other. The auction format can be either first-price or scoring. Under both auction formats, each firm submits a cash bid $b_i \in [0, r]$ and an intended quality $q_i \geq \underline{q} > 0$, where r is the reserve price and $\underline{q}$ is the minimum quality requirement. Under a first-price auction, the firm with the lowest cash bid wins the contract, and receives a payment equal to its bid. Under a scoring auction, a subjective evaluation process assigns each bidder i a noisy quality evaluation $\widehat{q}_i$ whose distribution is increasing in intended quality q_i in the sense of first-order stochastic dominance. The firm with highest score $s_i = \widehat{q}_i / b_i$ wins the contract, and receives a payment equal to its bid.[1] At the end of each auction, the auctioneer makes public the cash bids and recorded quality scores of each firm. Importantly, firms don't observe their competitors' intended qualities, so monitoring is imperfect under scoring auctions.[2]

Our first set of results formalizes that bidding patterns in the MLIT auctions are difficult to reconcile with a competitive model. We show that, when firms compete and there is no noise in the evaluation of quality (i.e., $\widehat{q}_i = q_i$), scoring auctions lead to higher winning bids relative to first-price auctions. Intuitively, firms compete by providing higher quality under scoring auctions, leading to higher procurement costs and higher bids. Furthermore, while noisy quality evaluation may lead to some reduction in bids from bidders seeking to guarantee allocations, we show that winning bids after the policy change tend to be isolated, which is inconsistent with competition even under noisy allocation (Chassang et al., 2022).

Next, we study how changes in the auction format affect bidding behavior under collusion, and draw inferences about which incentive constraints were binding to firms participating in MLIT auctions. The main challenge is that scoring corresponds to a bundle of disruptions to cartel behavior: it distorts allocation away from the most efficient bidder; it introduces new quality deviations; and it introduces imperfect monitoring. To identify which channels explain our data, we leverage the fact that the introduction of scoring auctions was staggered.

[1] We use the scoring rule $s = \widehat{q}/b$, since this was the rule used by Japan's MLIT. However, our analysis and results don't depend on this particular scoring rule.

[2] This is true in our application: cash bids and recorded qualities were made public after each auction; bidder proposals were not made public.

This allows us to compare first-price auctions held in the pre period (i.e., prior to 2005) with first-price auctions held in the post period; and to compare first-price auctions held in the post period with scoring auctions held in the post period.

We proceed in two steps. First, we argue that incentive constraints with respect to price deviations were binding in first-price auctions. If these constraints were not binding, bidding patterns in first-price auctions before and after the policy change would be similar. In the data, the winning bid distribution in first-price auctions held in the pre period first-order stochastically dominates the winning bid distribution in first-price auctions held in the post period. This implies that incentive constraints for price deviations were binding, and that the introduction of scoring auctions reduced the value of cooperation for the cartel.

In the second step, we argue that incentive constraints with respect to quality deviations were made binding by imperfect monitoring. We show that, if either firms' intended quality was observable (i.e. monitoring was perfect), or if incentive constraints with respect to quality deviations were not binding, then a cartel able to sustain high prices under first-price auctions would also be able to sustain the same high prices under scoring auctions. In the data, however, the winning bid distribution in first-price auctions held in the post period first-order stochastically dominates the winning bid distribution in scoring auctions held in the post period. This implies that scoring auctions introduced imperfect monitoring which made incentive constraints with respect to quality deviations binding.

Our results have implications for the design of effective procurement mechanisms. Procurement agencies are sometimes hesitant to use scoring auctions to allocate contracts, fearing that this may open the door to corruption between bidders and the auctioneer.[3] Our results show that scoring auctions may be an appropriate choice when collusion among bidders, rather than corruption between bidders and auctioneer, is the main concern.[4] In

[3]Burguet and Che (2004) study the impact of corruption on the outcomes of scoring auctions.

[4]Features of Japan's MLIT's implementation of scoring likely limited the scope for corruption. The task of assigning quality scores to bids was performed by a committee of engineers whose identity was unknown for the length of their terms.

addition, our results suggest that efforts to increase transparency, for instance by ensuring that scores are assigned using an explicit formula on the basis of objective information alone, or by publishing details of firms' proposals, may facilitate collusion between bidders and increase prices.

Related literature. This paper builds on a large theoretical literature studying the impact of imperfect monitoring in repeated games (Green and Porter, 1984, Abreu et al., 1986, 1990, Fudenberg et al., 1994). Our main inference result compares the enforceability of high bids under first-price and scoring auctions. A related result by Kandori (1992) shows that worsening the monitoring structure by garbling public signals reduces the set of enforceable actions, and shrinks the set of sustainable values. This comparative static result cannot be directly applied because it requires keeping stage-game payoffs unchanged. In contrast, changing the auction format from first-price to scoring changes both the monitoring structure and the stage game. Hence, it does not immediately follow from Kandori (1992) that scoring always reduces a cartel's ability to collude. Our theory identifies necessary conditions for collusion to be harder to sustain in scoring auctions than in first-price auctions.

A substantial body of work has studied the effect of scoring on bidding and auction outcomes. Che (1993) studies optimal scoring rules when bidders' types are one-dimensional. Asker and Cantillon (2008, 2010) consider more general environments in which bidders' types may vary along multiple dimensions. Takahashi (2018) studies empirically bidders' strategies in design-build auctions in which bidders face uncertainty over the subjective evaluation of their proposal. All of these papers assume that bidders behave competitively. We add to this literature by studying theoretically and empirically how scoring rule affects auction outcomes under collusion.

Several papers have studied auction design in the presence of collusion. Pavlov (2008), Che and Kim (2006, 2009) and Abdulkadiroglu and Chung (2003) show that collusion may be successfully limited by auction formats that require bidders to make ex ante payments. Che et al. (2018) study optimal auction design when collusive bidders don't have deep pockets and cannot make transfers. Chassang and Ortner (2019) argue that procurement auctions with price floors may reduce bidder collusion, by making price wars and entry deterrence strategies less effective. Starc and Wollmann (2022) study the extent to which entry can limit collusion. Our work complements these papers by showing that auction formats that introduce imperfect monitoring can be effective in curbing bidder collusion.

Finally, our paper contributes to a body of work that studies whether different theories of collusion provide a good description of actual collusive behavior. Porter (1983) and Ellison (1994) test whether pricing patterns from the Joint Executive Committee are consistent with the classic repeated game models of Green and Porter (1984) and Rotemberg and Saloner (1986). Wang (2009) tests for the presence of Edgeworth cycles (Maskin and Tirole, 2001) in retail gasoline markets. Kawai et al. (2023a) study bidding behavior by a detected cartel who used a mediator to sustain collusion.[5]

The rest of the paper is organized as follows. Section 2 provides institutional background, and describes the data, as well as motivating findings. Section 3 presents our theoretical framework. Section 4 argues that the data we have just shown is difficult to rationalize with either a competitive, or a perfectly collusive model. Section 5 characterizes cartel behavior under first-price auctions and scoring auctions. Section 6 formalizes our inference strategy by providing conditions under which cartel discipline becomes easier or harder to achieve before and after the policy change, and across auction formats. Section 7 draws inferences from MLIT auction data and establishes that: (i) cartel discipline was binding; (ii) maintaining cartel discipline was made more difficult by the introduction of scoring; (iii) although scoring is effectively a bundle of policies, part of the effect is due to imperfect monitoring challenges associated with subjective scores. Section 8 concludes. The Appendix collects proofs and

[5] Also related is the literature on cartel detection (Hendricks and Porter, 1988, Baldwin et al., 1997, Porter and Zona, 1993, 1999, Conley and Decarolis, 2016, Schurter, 2017, Chassang et al., 2022, Kawai and Nakabayashi, 2022, Kawai et al., 2023b). See Porter (2005), Harrington (2008), Asker and Nocke (2021), Hortaçsu and Perrigne (2021) for reviews of the literature.

further empirical analysis.

2 Institutional Background and Baseline Facts

2.1 MLIT Auctions

The Ministry of Land, Infrastructure and Transportation (MLIT) is the largest purchaser of construction projects in Japan, letting about 15,000 auctions worth about 1.7 trillion yen per year. The range of projects includes projects such as building bridges, tunnels, and large structures, as well as relatively small projects such as road maintenance and landscaping.

Introduction of Scoring Auctions. Prior to 2005, all but a handful of auctions were let using a first-price sealed-bid auction format. Under the first-price sealed-bid format, each bidder submits a cash bid, the lowest bidder wins and is paid its own price, subject to meeting a secret reserve price.

Starting mainly in 2005, the MLIT started changing the auction format to scoring auctions, first for large projects and subsequently for almost all projects, including small ones. In a scoring auction, each bidder submits a cash bid and a proposal. The proposal is then evaluated by the MLIT and given a numerical grade. For all projects, the numerical grade is normalized so that a proposal that meets, but does not exceed standard requirements, receives a numerical grade of 100. A proposal that exceeds standard requirements on certain dimensions may receive additional points. For example, for a road pavement contract, the auctioneer may allocate several points for efforts to mitigate disruption to traffic. Depending on how far the proposal goes beyond meeting the standard requirements, a proposal may receive any point between 0 and the maximum number of points allocated to that dimension. A proposal that does not meet standard requirements is disqualified. Typically, there are several dimensions along which a proposal is evaluated.

The maximum number of points a proposal can attain has been increasing over time.

Before October 2005, the maximum number of points was capped at 110. The cap was increased to 130 in October 2005, and then again to 170 at the beginning of 2007. The points allocated to each dimension are announced to the bidders beforehand. Anecdotally, however, there is residual uncertainty regarding the mapping between a given proposal and the final numerical grade.

In the scoring auctions used by the MLIT, the score s_i of bidder i is determined as follows:

$$s_i = \frac{q_i}{b_i}.$$

The score is increasing in the proposal quality q_i and decreasing in the cash bid b_i. The proposal quality q_i is not known to the bidder when submitting b_i. The project is allocated to the bidder with the highest score s_i among those who meet the reserve price.

Reserve prices. The MLIT sets reserve prices by summing the expected cost of all of the required inputs. The required inputs for projects let by scoring auctions are assessed on the basis of what would be needed to fulfill standard requirements, i.e., a proposal that would receive a numerical grade of 100. Importantly, the standard requirements for projects let in scoring auctions were set at the same level as those in first-price sealed bid auctions. Consequently, MLIT did not set different reserve prices for projects let as first-price sealed bid and scoring. This justifies comparing normalized bids (raw bids divided by the reserve price) auction formats to assess the impact of design changes. We show below that this is borne our in the data: for the subset of recurring auctions (i.e. auctions held annually with a fixed job description), reserve prices are unaffected by changes in the auction format.

Bidder participation. In addition to the introduction of scoring auctions, the MLIT introduced another notable change during our sample period regarding the eligibility of firms to participate in the auctions. Prior to 2005, for the majority of auctions, the bidders needed to be invited in order to participate. Around 2005, the MLIT began allowing all qualified bidders to participate. By the beginning of 2007, participation was open to all qualified bidders for most auctions.[6]

2.2 Auction Formats and Winning Bids Over Time

Figure 1 illustrates how the MLIT changed its procurement mechanism over time. For auctions held in each of Japan's nine regions, the figure plots the fraction of auctions that had a scoring format (solid line, right axis) and the fraction of open auctions (long dash, right axis). We also plot the normalized winning bid, i.e. the winning bid as a fraction of the reserve price (short dash, left axis). In each of the regions, we find a sharp increase in the share of scoring auctions and open auctions from around the end of 2005. The figure also shows a drop in the winning bid around the same time scoring and open auctions are introduced.

While the introduction of scoring auctions and the expansion of open participation took place around the same time, for the set of relatively large auctions, participation had been open to all qualified bidders by the beginning of our sample. Figure 2 is a time series plot of the share of scoring auctions, the share of open auctions, and the normalized winning bids for the set of auctions with a reserve price greater than 200 million yen. Figure 2 shows that open auctions were introduced much earlier than scoring auctions in most of the 9 regions. In particular, in six regions, the share of open auctions was already close to 100% at the beginning of our sample. The figure also shows that the decrease in the average winning bid occurs around the same time that scoring auctions are introduced. Lastly, a comparison of Figures 1 and 2 shows that scoring auctions were introduced earlier for large auctions. For example, by the middle of 2006, almost all of projects with a reserve price above 200 million yen were let let using scoring auctions. This reflects the fact that the MLIT introduced

[6]Even for open auctions, participation is not completely unrestricted. The MLIT maintained a form of market segmentation in which large firms are restricted from participating in small auctions and vice versa. For a more detailed description, see e.g., Asai et al. (2021).

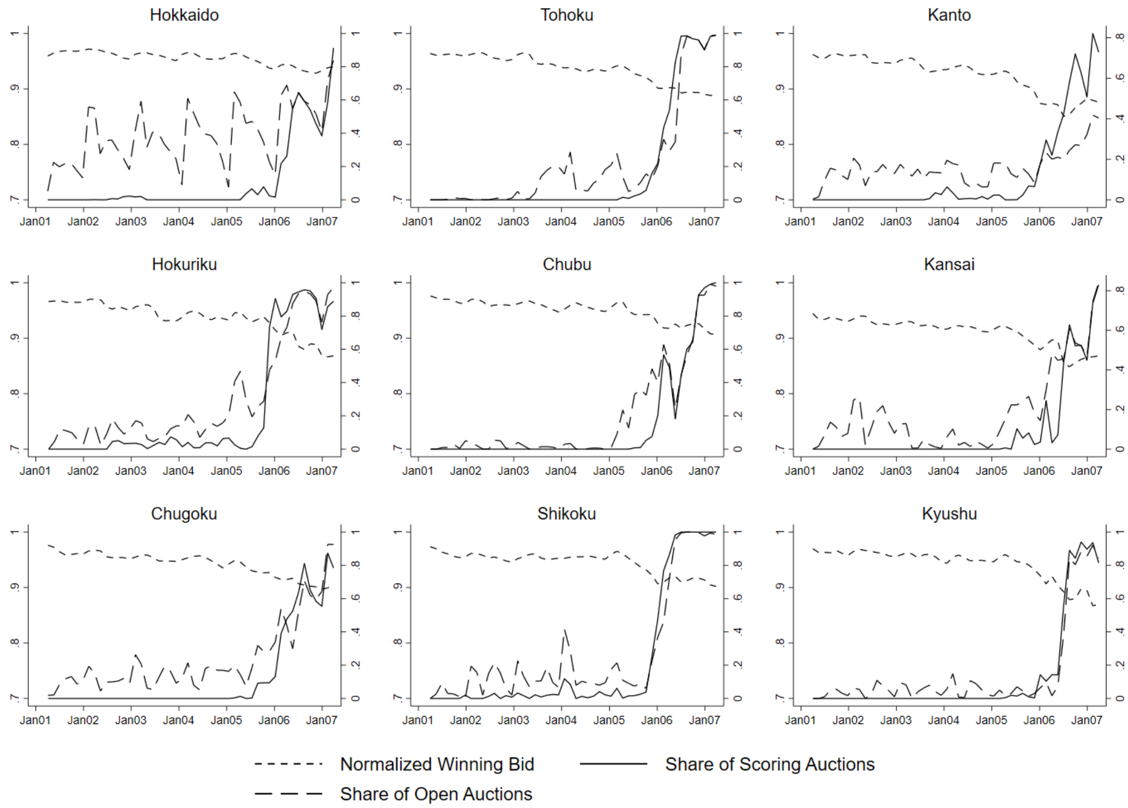

Figure 1: Evolution of winning bids as a fraction of reserve price (short dash, left axis), fraction of scoring auctions (solid line, right axis) and fraction of open auctions (long dash, right axis).

scoring auctions first for large projects, subsequently adopting it for smaller projects as well.

2.3 Summary Statistics

Table 1 reports the summary statistics of key variables by auction format. For first-price sealed-bid auctions, the average reserve price is about 100 million yen and the average winning bid is 94.7% of the reserve price. For scoring auctions, the average reserve is almost twice as large, at 197 million yen. The difference in the reserve price reflects the fact that the MLIT implemented scoring auctions first for larger projects and subsequently for all auctions, including smaller auctions. Large auctions are thus more heavily sampled among

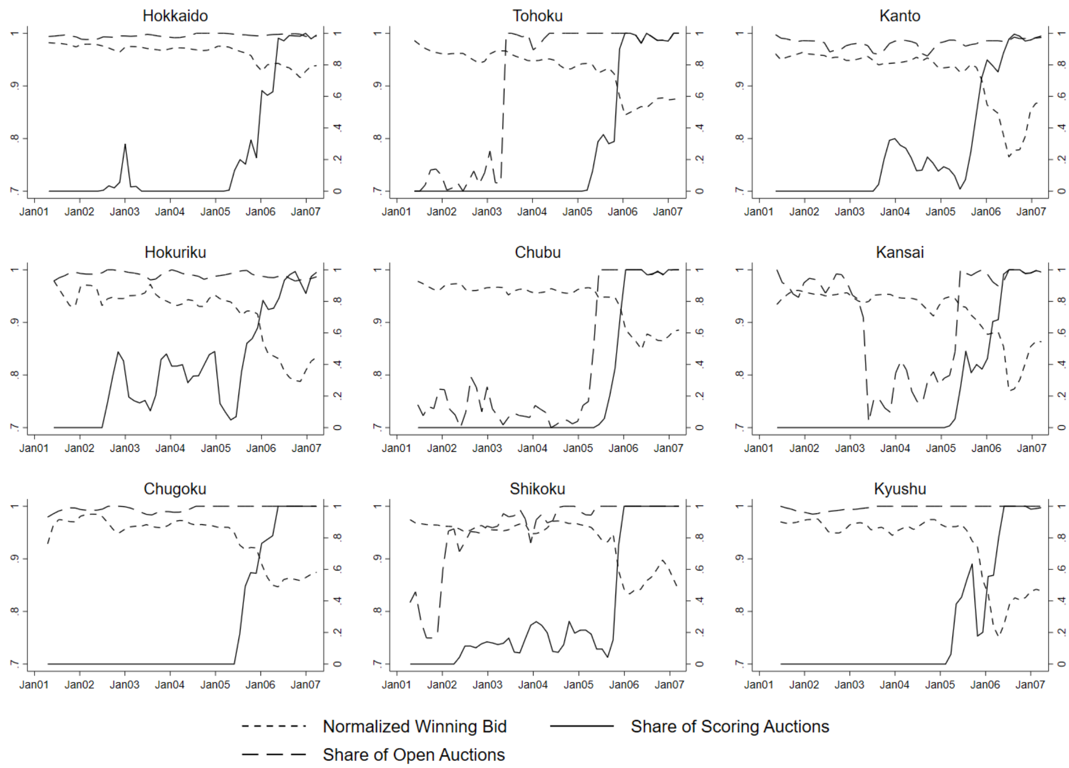

Figure 2: Evolution of winning bids as a fraction of reserve price (red, left axis), fraction of scoring auctions (blue, right axis) and fraction of open auctions (green, right axis), for auctions with a reserve price larger that 200 million yens.

scoring auctions.

The second row reports the average normalized winning bid. The average normalized winning bid for first price auctions is 94.7% and it is 89.5% for scoring auctions. The fact that the normalized winning bid is lower for scoring auctions is consistent with the hypothesis that collusion is harder to maintain under scoring auctions. The third row reports the second lowest cash bid.[7]

Another difference between first price auctions and scoring auctions is that the number of bidders is higher at 9.54 for first-price auctions than for scoring auctions at 7.52. This difference can be partly explained by reduction in complementary bidding. In a cartelized

[7] The second lowest cash bidder is the marginal bidder in first price auction. The second lowest cash bidder may not be the marginal bidder in a scoring auction.

	First Price Auctions			Scoring Auctions		
	Mean	SD	Obs.	Mean	SD	Obs.
Reserve (mil. Yen)	102.08	270.31	83116	197.03	423.38	11618
Winning Bid	0.947	0.071	83116	0.895	0.093	11618
Marginal Bid	0.967	0.059	79980	0.932	0.088	10475
# Bidders	9.54	2.78	83116	7.52	4.97	11618
# New Bidders	0.44	1.22	63391	0.28	0.87	11616
# Long-run Bidders	7.92	3.45	83116	6.59	4.80	11618
Winner's Quality	–	–	–	111.77	9.31	11231
Open Auctions	0.148	0.355	83116	0.879	0.327	11618

Some observations are missing information on the quality of the proposals. That is why the number of observations is smaller (11231) for the winning quality.

Table 1: Summary Statistics – all auctions

auction, firms that have no intention of winning participate to create the false impression of competition. Only serious bidders participate in a competitive auction, however, which can decrease the number of bidders. The simultaneous decrease in the winning bid and the number of bidders is consistent with more competition under scoring auctions.

Table 1 also reports the type of bidders that participate in the auctions. New bidders are those who have not participated in any auction for more than a year, and they account for about 4.6% of the bidders (0.44/9.54) in first-price auctions and 3.7% in scoring auctions. This number is not computed for auctions let in the first year of the sample. Long-run bidders are firms that are in the top quartile in terms of the number of auctions in which a firm participates over the entire sample. These firms account for about 83% (7.92/9.54) and 88% of bidders. By and large, the set of bidders that participate in scoring auctions are the same set of firms that participates in first-price auctions.

Table 2 reports summary statistics for the set of auctions with a reserve price above 200 million yen. This set corresponds to the sample of auctions in Figure 2. We find that scoring auctions have a higher average reserve price (506 million yen) than first price auctions (465 million yen) for the same reason as before: larger auctions are more heavily sampled among

	First Price Auctions			Scoring Auctions		
	Mean	SD	Obs.	Mean	SD	Obs.
Reserve (mil. Yen)	465.02	736.30	8536	506.51	752.88	2959
Winning Bid	0.960	0.054	8536	0.880	0.108	2959
Marginal Bid	0.976	0.044	8316	0.909	0.104	2845
# Bidders	9.42	3.49	8536	8.44	4.97	2959
# New Bidders	0.09	0.28	6184	0.08	0.28	2959
# Long-run Bidders	8.17	3.80	8536	7.43	4.93	2959
Winning Quality	–	–	–	113.58	12.23	2782
Open Auctions	0.762	0.426	8536	1.000	0.066	2959

Table 2: Summary Statistics – auctions with a reserve price above 200 million yens.

scoring auctions.

The average normalized winning bid is 96% for first-price auctions and it is about 8% lower, at 88%, for scoring auctions. Similarly, the second lowest cash bid decreases by about 7%. The number of new bidders is lower in this subsample than in the full sample, at 0.09 and 0.28. The number of long-run bidders, on the other hand, is higher, accounting for about 87% (8.17/9.42) and 88% (7.43/8.44) of the bidders. The share of open auctions is about 76% for first price auctions and close to 100% for scoring auctions.

2.4 Effects Are Not Driven by Changes in Reserve Prices

We now provide evidence that the MLIT did not change how they set the reserve price between first price and scoring auctions. In order to do this, for this subsection only, we focus on the sample of recurring auctions. We define recurring auctions as those that are let every fiscal year by the same branch office of the MLIT with the same project name. An example is the set of auctions let for the "management of Managawa dam flowerbed." Every fiscal year, the joint management office of the Kuzuryu river dam invites firms to bid on the upkeep of the flowerbed at the site of the dam. About 85% of recurring auctions are for maintenance projects.

FY	Reserve	Scoring	Win Bid	Marginal	# Bids	Previous	Obs.
2001	47.6 mil	0.00%	97.05%	98.26%	9.43	NA	758
2002	47.4 mil	0.00%	96.70%	98.07%	9.63	55.48%	758
2003	48.3 mil	0.00%	96.23%	97.54%	9.43	57.06%	758
2004	48.9 mil	0.00%	95.90%	97.55%	9.07	57.28%	758
2005	48.0 mil	0.66%	95.60%	97.24%	8.97	57.06%	758
2006	47.0 mil	44.99%	92.71%	95.92%	6.92	60.09%	758

Note: Reserve is in millions of JPY. Marginal is the bid of the marginal (second lowest bidder in FPSB, second highest bidder in Scoring) bidder. Previous is the percentage of participating bidders who also participated in the previous auction. The number of observations is lower for Marginal because there are some auctions with a single bidder.

Table 3: Summary of Recurring Auctions

Table 3 reports the summary statistic of recurring auctions by year. These auctions tend to be smaller, with average reserve price less than 50 million JPY in all years. Because these auctions are relatively small in size, the introduction of scoring auctions did not happen until 2006. Before 2006, the fraction of scoring auctions is close to 0%, and in 2006, it increases to about 45%. Notably, we see no increases in the average reserve price in 2006 when a little less than half of these auctions were let using scoring auctions. Moreover, we cannot reject the null that the distribution of the reserve price is the same between 2005 and 2006 using a Kolmogorov-Smirnov test (p=0.722). The fact that the reserve price is relatively stable across all years is consistent with the MLIT's policy of keeping the reserve price the same for first price and scoring auctions.[8]

Table 3 also reports the winning bid as a percentage of the reserve price, the number of bidders, and the percentage of bidders who participated in the previous auction. For example, in 2002, we find that 55.48% of the bidders participated in the same auction one year ago. The fraction of previous bidders is between 55% and 60% in all years. The normalized winning bid is above 95% until 2005, dropping to 92.7% in 2006.

Figure 3 displays the cumulative distribution function of normalized winning bids (left

[8]Note that reserve prices vary year to year at the individual project level, even for recurrent auctions. It is the distribution of reserve prices that is stable for recurrent auctions.

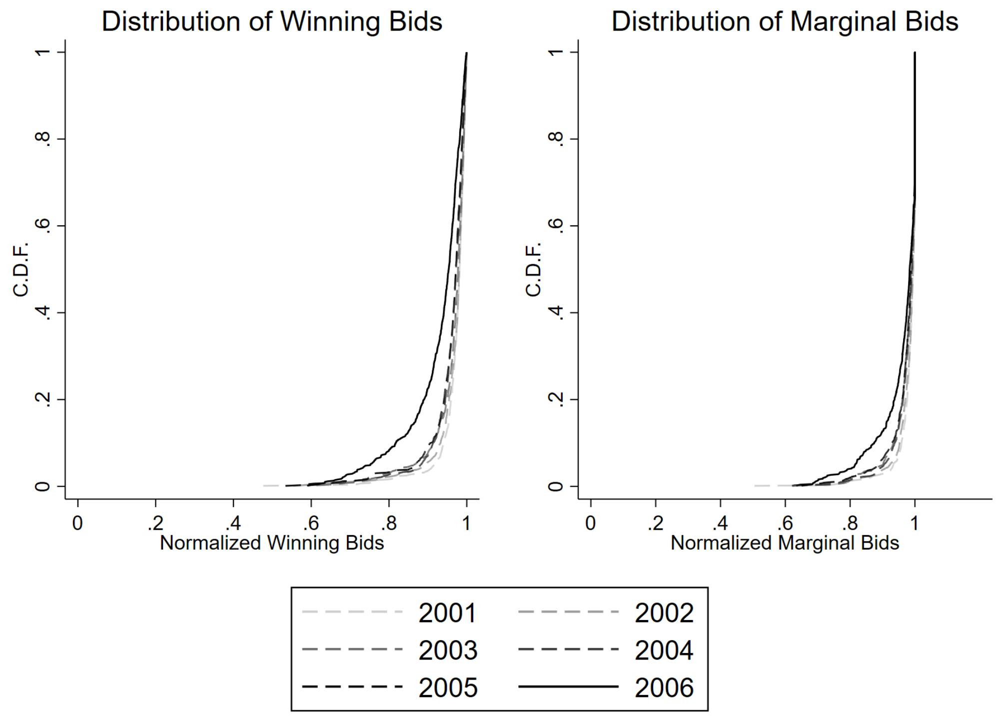

Figure 3: CDF of winning bids and marginal bid for recurring auctions.

panel) and the second lowest bids (right panel) in the subsample of recurring auctions for each year. The drop in normalized winning bids holds within recurrent auctions. This implies that the price drop illustrated in Figure 2 was not driven by a change in the distribution of reserve prices.

3 Framework

The point of our theoretical framework is to draw inferences from observed bidding on which frictions limit the extent of cartel behavior. This section describes the baseline game induced by first-price and scoring auctions. Section 4 clarifies that in competitive environments the introduction of scoring should increase rather than decrease prices. Section 6 models cartel discipline and delineates inference on which self-enforcement constraints bind.

Repeated procurement. At each period $t \in \mathbb{N}$, a buyer procures a single item through an auction from a group of firms $I = \{1, ..., n\}$. Each firm $i \in I$ can produce a good of quality $q_i \in Q \subset [\underline{q}, \overline{q}]$ (with $\overline{q} > \underline{q} > 0$) at a cost $c(q_i, \theta_i) \geq 0$ that depends on each firm i's type $\theta_i \in [0, 1]$ and is strictly increasing in both q and θ. Cost function $c(q, \theta)$ is assumed to satisfy increasing differences: for all $q' > q$, $c(q', \theta) - c(q, \theta)$ is increasing in θ. The set of qualities Q is finite, with $\underline{q}, \overline{q} \in Q$.

We assume that in each period t, the profile of firms' types $\theta_t \equiv (\theta_{i,t})_{i \in I}$ is drawn independently from past outcomes from a symmetric joint c.d.f. F with $\mathsf{supp}\, F \subset [0, 1]^I$.

Auction formats. Each period, a contract is allocated using one of two auction formats: a first price auction (FPA) or a scoring auction (SA).

Under both auction formats, each firm i submits a cash bid $b_i \in [0, r]$ (where r is the reserve price, normalized to $r = 1$) to be paid to i if they win, as well as a proposed quality $q_i \in Q$, which is noisily evaluated as $\widehat{q}_i \in Q$ by the auctioneer. We assume that, for each $i \in I$, the distribution of $\widehat{q}_i$ depends only on q_i. We further assume that the associated distribution $\gamma(\widehat{q}_i | q_i) \in [0, 1]$ is increasing in q_i in the sense of first-order stochastic dominance. Lastly, we assume that for all $q_i \in Q$, $\gamma(\cdot | q_i)$ has full support over Q. For each $\widehat{q} = (\widehat{q}_i)_{i \in I}$ and $q = (q_i)_{i \in I}$, we use $\mu(\widehat{q} | q) = \prod_i \gamma(\widehat{q}_i | q_i)$ to denote the probability of observing a profile of recorded qualities $\widehat{q}$ when the profile of firms' proposed qualities is q.

The two auction formats differ only in their allocation rule. Let us denote by $x_i \in \{0, 1\}$ whether bidder i wins the procurement contract or not. Under FPA, the bidder with the lowest cash bid wins the auction:

$$x_i \equiv \begin{cases} 1 & \text{if} \quad b_i < \wedge b_{-i} \\ 0 & \text{if} \quad b_i > \wedge b_{-i} \end{cases} .$$

where $\wedge b_{-i} \equiv \min\{b_j, j \in I \setminus i\}$ denotes the most competitive bid from bidders other than i.

Under SA, the bidder with the highest evaluated score s_i defined as $\frac{\widehat{q}_i}{b_i}$ wins the auction:

$$x_i \equiv \begin{cases} 1 & \text{if} \quad s_i > \vee s_{-i} \\ 0 & \text{if} \quad s_i < \vee s_{-i} \end{cases}.$$

where $\vee s_{-i} \equiv \max\{s_j, j \in I \setminus i\}$ denotes the most competitive score associated with bidders other than i.[9]

We note that, under both auction formats, bidders face the same minimum quality requirement $\underline{q}$. This is consistent with our empirical application: the MLIT didn't change minimum quality requirements when scoring auctions were introduced.[10] As was confirmed in Section 2 using recurring auctions, the way reserve prices were set also didn't change following the introduction of scoring.

Information. Consistent with our application, we assume that in each period t, after the auction takes place, bids $b_t \equiv (b_{i,t})_{i \in I}$ and *evaluated* qualities $\widehat{q}_t \equiv (\widehat{q}_{i,t})$ are publicly observed by all bidders. Actual quality submissions $q_t \equiv (q_{i,t})_{i \in I}$ are not observed.

We assume that the type profile $\theta_t = (\theta_{i,t})_{i \in I}$, which captures both project and bidder characteristics, is publicly observable at the beginning of period t, before bidding occurs: there is complete information about costs. This assumption makes the model tractable. Note that assuming complete information about costs is not unreasonable in our setting since firms are frequent participants in the same markets, likely to be informed of one another's circumstances. Uncertainty over future realizations of type profile θ reflects variation in firms' circumstances, as well as uncertainty over auction characteristics and fit with individual firms' capabilities.

[9]We follow Athey and Bagwell (2001) and let bidders jointly determine the allocation in case of ties. Formally, each bidder i submits a number $\gamma_i \in \mathbb{R}$. In case of ties, the contract is allocated with equal probability to the tied bidders that submit the lowest bid, and the lowest tie-break number γ.

[10]For example, see "Current state of scoring auctions" (National Institute for Land and Infrastructure Management, 2005) or Section 1-2 of the 2013 operational guideline issued by the MLIT.

Payoffs and costly transfers between firms. Firm i's payoff from the auction is

$$u_{i,t} = x_{i,t} \times (b_{i,t} - c(q_{i,t}, \theta_{i,t}))$$

In addition, firms can make costly monetary transfers between one another at the end of each period, after the outcome of the auction is realized. Let $T_{i,t}$ denote the aggregate net transfer received by firm i in period t. Transfers must be such that, for all $t \in \mathbb{N}$, $\sum_{i \in I} T_{i,t} = 0$. We assume that sending transfers is costly, with a loss rate $\lambda \geq 0$, so that, including transfers, firm i's net payoff in period t is

$$u_{i,t}^T \equiv x_{i,t}(b_{i,t} - c(q_{i,t}, \theta_{i,t})) + T_{i,t}(1 + \lambda \mathbf{1}_{T_{i,t}<0}).$$

Firms discount future payoffs with common discount factor $\delta \in [0, 1)$.[11]

Policy variation. Let us denote by $f_t \in \{\text{FPA}, \text{SA}\}$ the auction format selected in period t. We idealize the variation in auction format in the data and assume that:

- In all periods t before the policy change (pre periods), the auction format f_t was $f_t = \text{FPA}$. Firms believe that a regime switch corresponding to the introduction of scoring auctions takes place with fixed probability $\alpha \in (0, 1)$.

- In all periods t after the regime switch (post periods), the auction format f_t is selected from $\{\text{FPA}, \text{SA}\}$ with a full support distribution that depends only on observable auction characteristics. This allows the choice of the auction format to depend on project characteristics, such as the date, the region, the type of work being procured, the reserve price, and so on.

[11] We note that many known cartels used monetary transfers (Pesendorfer, 2000, Asker, 2010, Harrington and Skrzypacz, 2011). In practice, cartel members can make transfers by sub-contracting each other. In addition, in the equilibrium that we focus on in Section 5, costly on-path transfers can be thought of as a metaphor for costly on-path punishments.

This allows us to perform the following comparisons: (i) comparisons between FPA auctions in the pre vs. post periods; and (ii) comparisons between FPA vs. SA auctions in the post periods. Comparison (i) keeps fixed the auction format (and associated stage game) and identifies the impact of changing continuation values. Comparison (ii) keeps fixed the set of possible continuation values used to enforce bidding behavior, and contrasts different auction formats.

Strategies and solution concepts. Let us denote by $h_{t^-} \equiv (\theta_s, f_s, b_s, \widehat{q}_s, T_s, \theta_t, f_t)_{s<t}$ and $h_{t^+} \equiv (\theta_s, f_s, b_s, \widehat{q}_s, T_s, \theta_t, f_t, b_t, \widehat{q}_t)_{s<t}$ the public history of play in period t before and after auction outcomes are realized.

A public strategy σ_i for firm i is a mapping from public histories to bidding and transfer actions:

$$\begin{aligned} \sigma_i : \quad & h_{t^-} \mapsto (b_{i,t}, q_{i,t}) \\ & h_{t^+} \mapsto T_{i,t} \end{aligned}$$

Throughout, we say that a strategy profile $\sigma = (\sigma_i)_{i \in I}$ is *competitive* if firms play a Nash equilibrium in weakly undominated strategies of the stage game in every period. We note that no transfers are made under a competitive strategy profile.

4 The Impact of Scoring under Competition

This section argues that the core takeaway from Section 2 – scoring led to a reduction in winning bids – goes directly against the intuitive impact of scoring under competition.

4.1 Precise Quality Evaluations

To help build intuition around the facts presented in Section 2, we show that scoring increases competitive prices when quality is precisely evaluated. Within this subsection, we assume that quality is precisely evaluated: for all q_i, distribution $\gamma(\cdot|q_i)$ has a unit mass at q_i.

Proposition 1 (scoring increases prices (I))**.** *Under both FPA and SA auctions, the stage game admits a unique Nash equilibrium in weakly undominated strategies. The contract is allocated to the most efficient bidder, i.e. the bidder with minimum cost type* $\min\{\theta_i, i \in I\}$*.*

For $f \in \{FPA, SA\}$*, let* $b_f^{(1)}(\theta)$ *denote the stage game Nash winning bid at type profile* θ *under auction format* f*. We have that, for all* θ*,* $b_{SA}^{(1)}(\theta) \geq b_{FPA}^{(1)}(\theta)$*.*

In words, fixing the bidders' types, winning bids under scoring auctions are weakly larger than winning bids under first-price auctions. The intuition behind this result is straightforward: scoring auctions induce bidders to provide higher quality, increasing procurement costs and winning bids. An immediate Corollary of Proposition 1 is that, under competition, the policy change in Japan should have led to higher winning bids. The data in Section 2 is at odds with this prediction.

Corollary 1. *Under the assumption that quality is precisely evaluated and that firms are competitive:*

(i) the distribution of winning bids in the post *period first-order stochastically dominates the distribution of winning bids in the* pre *period;*

(ii) in the post *period, the distribution of winning bids from SA first-order stochastically dominates the distribution of winning bids from FPA.*

4.2 Imprecise Quality Evaluations

We now allow for noisy quality evaluations. In contrast to the case of precise evaluation, it is no longer the case that competitive behavior will lead to efficient allocation under SA: noise in quality evaluation may lead the contract to be allocated away from the lowest cost firm. As a result, competitive firms may adjust their bidding to improve the odds of correct allocation, potentially reducing prices under SA.

An example. A simple example illustrates how noisy quality evaluation can reduce the winning bid. Assume for simplicity that there are two repeated bidders, firm 1 and firm 2, whose costs (c_1, c_2) alternately take values (c_L, c_H) and (c_H, c_L), with $c_L < c_H$. Firms don't actually choose quality – it is set at $\underline{q}$. Under the FPA, in any Nash equilibrium, both bidders bid c_H, and the good is allocated to the lowest cost bidder (via the choice of tie-breaking numbers as in Athey and Bagwell (2001)). Now consider the case of scoring, and assume that quality evaluations take the form $\widehat{q}_i = \underline{q} + \delta_i \varepsilon$ with $\varepsilon > 0$ and δ_i a Rademacher random variable taking values -1 and $+1$ with equal probabilities. For ϵ small enough, the highest cost bidder will bid c_H while the lowest cost bidder will bid $c_H - 2\epsilon$ to ensure they win the contract.[12]

Still Proposition 1 extends as follows.

Proposition 2 (scoring increases prices (II))**.** *For $f \in \{FPA, SA\}$, let $b_f^{(2)}(\theta)$ denote the second lowest bid at type profile θ. Under any Nash equilibrium in weakly undominated strategies, $b_{SA}^{(2)}(\theta) \geq b_{FPA}^{(2)}(\theta)$.*

Note that a similar argument would apply under a perfect cartel (one not limited by coordination and discipline frictions): the highest cost bidders would all set bids equal to the reserve price regardless of the auction format. Altogether, this implies that the drop in bids associated with scoring is associated with imperfect cartel behavior. To draw further inferences, the next section introduces a model of imperfect cartel discipline.

5 Limits to Cartel Discipline

5.1 Model

We model cartel discipline using a repeated game framework.

[12]A similar argument holds in the context of a perfect cartel (one that isn't limited by coordination or discipline frictions). Under the FPA, the lowest cost cartel member would win at the reserve price r. Under the SA, for ϵ small, the lowest cost cartel member would win at $r - 2\epsilon$.

Solution concept. We focus on strategy profiles σ mapping public histories to price and quality bids (b, q) and transfers T such that

(i) Profile σ is a perfect public equilibrium (PPE, Fudenberg et al., 1994);

(ii) Behavior is stationary and symmetric on path: for any on-path histories h_t^- and h_t^+, bidding behavior $\sigma(h_{t^-})$ and transfer behavior $\sigma(h_{t^+})$ are independent of previous observables $(\theta_s, b_s, \widehat{q}_s, T_s)_{s<t}$ and invariant to a relabeling of players;

(iii) Off-path punishment is achieved through reversion to stage game Nash.

We refer to such equilibria as Stationary PPEs.

Let us denote by $\underline{V}$ the discounted cartel profits $\frac{1}{1-\delta}\mathbb{E}\left[\sum_{i\in i} u_i\right]$ under stage game Nash and by $\overline{V}$ the highest discounted cartel profits $\frac{1}{1-\delta}\mathbb{E}\left[\sum_{i\in i} u_i^T\right]$ sustainable under a Stationary PPE. Note that gross payoffs u_i can be used to evaluate surplus under stage game Nash since no transfers are used. Payoffs net of transfer costs u_i^T must be used to evaluate more general Stationary PPEs since transfers may happen on path.

Let $[x]^- = \max\{-x, 0\}$. Recall that, for any $\widehat{q} = (\widehat{q}_i)_{i\in I}$ and any $q = (q_i)_{i\in I}$, $\mu(\widehat{q}|q) = \prod_i \gamma(\widehat{q}_i|q_i)$ is the probability of observing recorded qualities $\widehat{q}$ when firms' intended qualities are q.

Lemma 1 (cartel optimal behavior)**.** *At any profile of types $\theta = (\theta_i)_{i\in I}$, cartel optimal bidding solves*

$$\max_{b,q} \sum_{i\in I} u_i(b, q, \theta_i) - \lambda K(b, q, \theta) \tag{P}$$

with

$$K(b,q,\theta) \equiv \min_{T:Q^I \mapsto [\underline{T},+\infty)^I} \mathbb{E}_\mu \left[\sum_{i\in I} [T_i(\widehat{q})]^- \right] \;\; s.t. \tag{K-IC}$$

$$\forall i \in I, \forall b_i' \neq b_i, \forall q_i', \quad u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) - u_i(b,q,\theta_i) \tag{IC-p}$$

$$\leq \sum_{\widehat{q}\in Q^I} \mu(\widehat{q}|q) T_i(\widehat{q})(1+\lambda \mathbf{1}_{T_i(\widehat{q})<0}) + \frac{\delta}{n}(\overline{V} - \underline{V})$$

$$\forall i \in I, \forall q_i' \neq q_i, \quad u_i(b, q_i', q_{-i}, \theta_i) - u_i(b,q,\theta_i) \tag{IC-q}$$

$$\leq \sum_{\widehat{q}\in Q^I} (\mu(\widehat{q}|q) - \mu(\widehat{q}|q_i', q_{-i})) T_i(\widehat{q})(1+\lambda \mathbf{1}_{T_i(\widehat{q})<0})$$

$$\forall \widehat{q} \in Q^I, \quad \sum_{i\in I} T_i(\widehat{q}) = 0. \tag{BB}$$

where $\underline{T} = -\frac{1}{1+\lambda}\frac{\delta}{n}(\overline{V} - \underline{V})$ *and, by convention,* $K(b,q,\theta) = +\infty$ *if* b, q *is such that there are no transfers* $T : Q^I \mapsto [\underline{T}, +\infty)^I$ *satisfying* (IC-p), (IC-q) *and* (BB).

Program (K-IC) captures the cost of enforcing a particular bidding profile (b,q) at a profile of types θ. It is computationally tractable: it minimizes a convex piece-wise linear function over convex piece-wise linear constraints.[13]

Constraints (IC-p) and (IC-q) capture the economics of cartel enforcement. In both cases, the flow payoff increase from deviating – either in bid and quality for (IC-p), or in quality alone for (IC-q) – must be bounded above by the loss in expected contemporaneous transfers and continuation values caused by the deviation.

(IC-p) speaks to deviations involving bids. Because bids are observable, such deviations lead to off-path histories, and can be punished using a surplus-destroying reversion to stage game Nash without reducing the on-path surplus. As a result, transfers only occur following equilibrium play. The difference in on-path continuation surplus $\overline{V}$ and off-path continuation surplus $\underline{V}$ provides dynamic incentives.

[13] We note that, since quality deviations don't affect the allocation when the auction format is FPA, (IC-q) does not bind under these auctions.

(IC-q) speaks to deviations involving quality alone. Because quality deviations are not observable, they lead to on-path histories. Under the assumption of Stationary PPEs this means that quality deviations don't affect future continuation play, and instead have to be dissuaded through contemporaneous transfers designed to be higher following histories more likely to occur following deviations. Because these transfers take place at on-path histories, they reduce on-path surplus. This is the additional effect of imperfect monitoring on enforcement constraints.

Two additional observations are useful: (i) in first-price auctions, flow payoffs from quality deviations are null, and (IC-q) is satisfied by any transfer scheme that depends on bids b and types θ alone; (ii) if intended quality q was observable, then (IC-q) would become identical to (IC-p), and the two constraints could be combined by allowing the case $b_i' = b_i$ in (IC-p).[14]

One can solve for the highest sustainable cartel surplus using a fixed-point approach. For any on-path continuation value $V \geq \underline{V}$ and any vector of types $\theta = (\theta_i)_{i \in I}$, let $(b_V(\theta), q_V(\theta))$ denote the solution to program (P) when $\overline{V}$ is replaced by V. Define the mapping $\Phi : \mathbb{R}_+ \to \mathbb{R}_+$ as

$$\Phi(V) = \mathbb{E}_F \left[\sum_{i \in I} u_i(b_V(\theta), q_V(\theta), \theta) - K(b_V(\theta), q_V(\theta), \theta) \right] + \delta V.$$

Then, $\overline{V}$ is the largest fixed-point of Φ. It can be computed by iteratively applying Φ to any upper bound V_0 on feasible values.

5.2 How scoring affects IC constraints

We now discuss how the introduction of scoring rules affects the incentive constraints faced by a cartel. We identify 4 different channels, three of which make collusion more difficult. The other can make collusion easier.

[14] Note that there is no discontinuity at the limit where quality assessments have full support but becomes arbitrarily precise. The cost of transfers following unlikely histories is then approximately equal to the difference in continuation values $\frac{1}{n}(\overline{V} - \underline{V})$.

Misallocation and pledgeable surplus. As we highlighted in Section 4, even when IC constraints (IC-p) and (IC-q) are slack, scoring can result in misallocation of the project to higher cost firms that reduce flow payoffs. This mechanically reduces cartel surplus $\overline{V}$.

Noisy evaluation also affect stage game Nash payoffs. When firms' costs are close, so that profits under the FPA are close to 0, noisy evaluations increase cartel profits under stage game Nash, thereby increasing $\underline{V}$.

In this particular configuration (i.e., slack constraints, and low FPA profits) scoring reduces the pledgeable surplus $\overline{V} - \underline{V}$, which limits collusion. Note that this effect exists even if quality evaluations are precise, or if there is no imperfect monitoring.

Price deviation temptation. When IC constraints are binding, scoring affects the left-hand side of (IC-p): undercutting another bidder's bid becomes less attractive under SA relative to FPA since the change in allocation now has probability less than 1. This facilitates collusion.

Quality deviation temptation. Under FPA there is no deviation temptation associated with quality deviations: evaluated quality does not affect the allocation. Under SA, quality choices can change the allocation creating a deviation temptation with respect to quality. This makes collusion more difficult.

Imperfect monitoring. Quality deviations are different from price deviations because quality choices are unobserved and evaluated quality has full support. As a result, quality deviations are deterred by costly transfers T on path. These reduce the surplus available to the cartel and makes collusion more difficult.

In the next section we attempt to sign the aggregate impact of these different mechanisms on enforceable bidding. Appendix A presents a simple, fully solved, example illustrating how scoring auctions can affect a cartel's ability to sustain collusive bidding.

6 Drawing Inferences about Cartel Discipline

We assess the impact of different mechanisms by comparing two different sets of auctions:

- Comparing pre and post FPAs allows us to identify the impact of scoring through its effect on pledgeable surplus $\overline{V} - \underline{V}$ alone.

- Comparing post FPAs and post SAs allows us to identify the impact of quality deviations and imperfect monitoring, keeping fixed pledgeable surplus.

We start by noting that if either incentive constraints for price deviations (IC-p) did not bind in the pre and post periods, or if values $\overline{V}$ and $\underline{V}$ remained unchanged across the two periods, then optimal collusive bidding under FPA would be the same in the pre and post periods. Hence, under either of these conditions, the distribution of winning bids under FPAs held in the pre and post periods should be identical. Our next result summarizes this.

Proposition 3. *If* (IC-p) *is not binding under FPA (pre and post), or if*

$$\overline{V}_{\mathsf{pre}} - \underline{V}_{\mathsf{pre}} = \overline{V}_{\mathsf{post}} - \underline{V}_{\mathsf{post}}$$

then winning bids under FPA in the pre and post periods have the same distribution.

In Section 7, we show that the c.d.f. of winning bids under FPAs held in the pre period first-order stochastically dominates the c.d.f. of winning bids under FPAs held in the post period. This implies that incentive constraints for price deviations were binding, and that the introduction of scoring reduced the cartel's ability to enforce collusive bids (i.e., $\overline{V}_{\mathsf{pre}} - \underline{V}_{\mathsf{pre}} > V_{\mathsf{post}} - \underline{V}_{\mathsf{post}}$).

Our next result highlights conditions under which the introduction of scoring does not lead to lower winning bids.

Proposition 4 (SA vs. FPA bids without imperfect monitoring)**.** *Suppose* $\delta(\overline{V} - \underline{V}) > 0$. *There exists* $\eta > 0$ *such that, if* $\max_{i,j} |c(\underline{q}, \theta_i) - c(\underline{q}, \theta_j)| < \eta$, *and if either of the following hold:*

(i) (IC-q) *is not binding (e.g. quality is fixed);*

(ii) quality is noisily evaluated but perfectly monitored;

(iii) transfers are costless (i.e., $\lambda = 0$*);*

then, there exists $\underline{b} < r$ *such that enforceability of a winning bid of* $b \in [\underline{b}, r]$ *under FPA implies enforceability of a winning bid of* b *under SA.*

To understand Proposition 4, note that, while scoring auctions may introduce imperfect monitoring, they also change the stage game that firms play. An important observation is that price defection by a cartel member (undercutting the winning bid) is less profitable under scoring auctions than under first-price auctions. Indeed, under FPA, a bidder who undercuts the winning bid wins the auction with probability 1. In contrast, under SA, undercutting the winning bid leads to a less than certain probability of winning when quality is noisily measured.[15] Hence, for scoring to reduce equilibrium winning bids under collusion, it must be that monitoring is imperfect under scoring, and that incentive constraints for quality deviations are binding. This explains points (i) and (ii) in Proposition 4.

To understand Proposition 4(iii), note that if transfers are costless (i.e., $\lambda = 0$), the cartel can costlessly deter quality deviations by making firms pay a large transfer whenever they receive a high quality score $\widehat{q}$: even if these transfers end up being paid on the equilibrium path, they don't destroy aggregate cartel surplus when $\lambda = 0$.

In Section 7, we show that the distribution of winning bids of first-price auctions held in the post period first-order stochastically dominates the distribution of winning bids of

[15]Building on this observation, Kawai et al. (2023a) and Ortner et al. (2023) study how mediation can help cartels sustain higher equilibrium profits.

scoring auctions held in the same period. According to our theory, this implies that there is imperfect monitoring, and that incentive constraints for quality deviations were binding.

Our last result highlights that scoring can only limit the cartel's ability to collude in a sweet-spot region where the cost of providing increased quality the noise in quality assessments are both intermediate.

Proposition 5 (A Goldilocks effect)**.** *Suppose* $\delta(\overline{V} - \underline{V}) > 0$*. There exists* $\eta > 0$ *and* $\epsilon > 0$ *such that, if* $\max_{i,j} |c(\underline{q}, \theta_i) - c(\underline{q}, \theta_j)| < \eta$*, and if*

$$\max_{q \neq q', \theta} |c(q, \theta) - c(q', \theta)| < \epsilon \text{ or } \min_{q \neq q', \theta} |c(q, \theta) - c(q', \theta)| > \frac{1}{\epsilon},$$

or if

$$\min_{q \neq q', \widehat{q}} |\ln \gamma(\widehat{q}|q) - \ln \gamma(\widehat{q}|q')| > \frac{1}{\epsilon} \text{ or } \max_{q \neq q', \widehat{q}} |\ln \gamma(\widehat{q}|q) - \ln \gamma(\widehat{q}|q')| < \epsilon,$$

then, there exists $\underline{b} < r$ *such that enforceability of a winning bid of* $b \in [\underline{b}, r]$ *under FPA implies enforceability of a winning bid of* b *under SA.*

Proposition 5 implies that, for scoring to limit collusion, two things must be true: (i) firms' cost of providing higher quality should be large, but not too large; and (ii) monitoring should be imperfect, but not arbitrarily noisy. To see why, note that if the cost of providing higher quality is very large, or if monitoring is too noisy, then quality incentive constraints will not bind; and so by Proposition 4, scoring would lead to higher equilibrium bids. Similarly, if the cost of providing higher quality is negligible, colluding firms can almost costlessly provide higher quality, and hence quality incentive constraints will not bind. Lastly, if monitoring is almost perfect, quality deviations can be deterred at almost zero cost by having firms pay a large transfer whenever they get a high recorded quality.

7 Empirical Findings

This section draws inferences from the data. Guided by the theory in the previous section, we focus on two comparisons: (i) first-price auctions held in the pre period vs. first-price auctions held in the post period; and (ii) scoring auctions held in the post period vs. first-price auctions held in the post period.

First-price auctions in pre and post periods. Proposition 3 shows that if incentive constraints for price deviations (IC-p) do not bind, or if the cartel's continuation values remain unchanged, then the distribution of winning bids should be the same for first-price auctions in the pre and post periods. We now show that this is not the case: the winning bid distribution for first-price auctions fell in terms of first-order stochastic dominance following the introduction of scoring auctions. The implication is that incentive constraints are binding, and that the introduction of scoring auctions reduced cartel continuation payoffs.

Figure 4 plots the c.d.f. of winning bids in first price auctions for groups of firms with different experience with scoring auctions in 2005. For each firm that bid in at least 10 auctions in 2005, we compute the proportion of auctions that were scoring auctions. We then divide the firms into those that had high exposure, moderate exposure, low exposure, and no exposure.[16] We find that, while there is a time trend affecting all firms, the distribution of winning bids changes significantly for 2005 and 2006 for firms with positive exposure. Moreover, the figure shows that firms with higher exposure experienced more significant decreases in 2005 and especially 2006.

Next, we estimate a regression of the following form in the sample of first-price auctions:

$$winning\_bid_{i,t} = \beta_p \mathsf{post}_{i,t} + \beta X_t + \delta_i + \epsilon_{i,t}, \tag{1}$$

[16] Slightly more than a half of all firms had no exposure. Among firms with positive exposure, we take the top 25% as high exposure, the next 50% as moderate exposure, and the bottom 25% as low exposure.

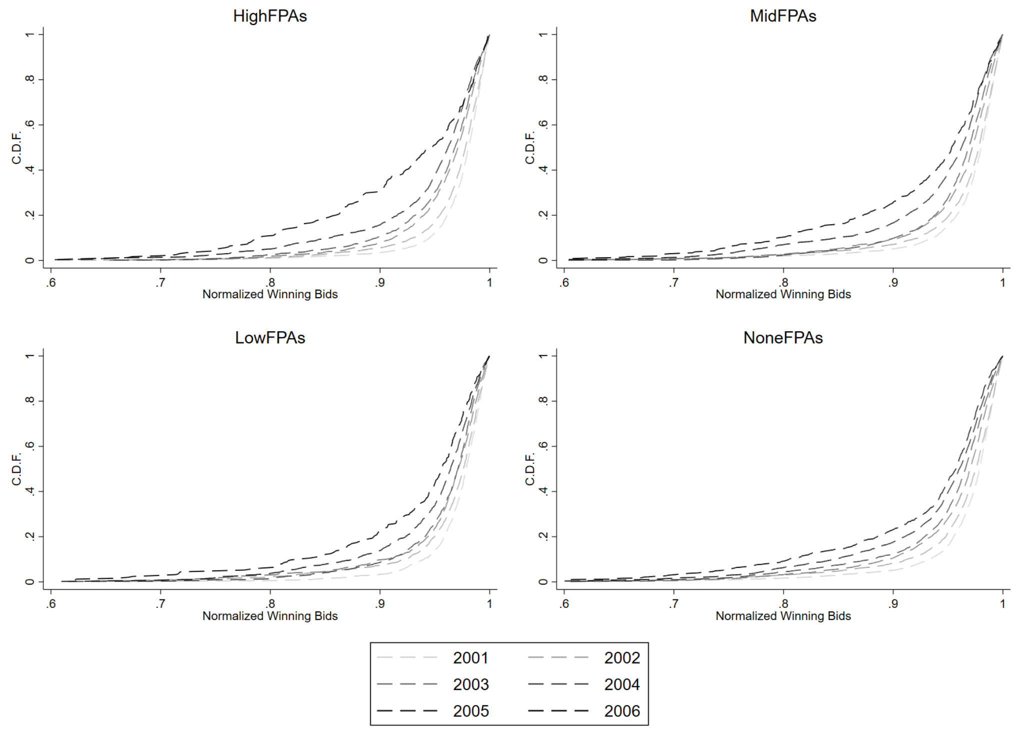

Figure 4: Distribution of normalized winning bids for first-price auctions. Top left panel corresponds to firms with high exposure to scoring auctions in 2005. Top right and bottom left panel correspond to those with moderate and low exposure, respectively. The bottom right panel corresponds to those with no exposure.

where the outcome variable is the bid of the winner as a fraction of the reserve price and $\mathsf{post}_{i,t}$ is a measure of firm i's exposure to scoring auctions. We define $\mathsf{post}_{i,t}$ as the proportion of scoring auctions that firm i participates in the 90 days or 180 days preceding the auction in question. We define this variable for firms that participate in at least 5 or 10 auctions during the preceding 90 or 180 days to reduce attenuation bias.[17] X_t is a vector of control variables and δ_i is a firm fixed effect. Unless otherwise noted, we include project category fixed effects and the full interaction of month and year dummies as controls in X_t. Project category fixed effects control for the type of project being let, such as road paving or maintenance.

Table 4 presents the estimates of regression (1). The first two columns correspond to the

[17] Measurement error in $\mathsf{post}_{i,t}$ leads to coefficient estimates that are biased towards zero.

	(1) Frac. 90	(2) Frac. 90	(3) Frac. 180	(4) Frac. 180
post	-0.0226***	-0.0277***	-0.0169***	-0.0157**
	(0.0041)	(0.0061)	(0.0040)	(0.0052)
N	31,015	17,606	48,201	34,109

*, **, and *** respectively denote significance at the 5%, 1%, and 0.1% levels.

Table 4: First Price Auctions in Pre and Post Periods

results when $\mathsf{post}_{i,t}$ is defined as the proportion of scoring auctions a firm participates within the 90 days preceding the auction in question. We use firms that participate in at least 5 auctions during the 90-day window for Column (1) and firms that participate in at least 10 auctions for column (2). Columns (3) and (4) report the results when we use the 180 day period prior to the auction to compute $\mathsf{post}_{i,t}$.

The results of Table 4 indicate that bidders that are exposed to scoring auctions have winning bids that are lower by between 1.57 and 2.77 percentage points, depending on the specification. The implication is that constraint (IC-p) was binding, and that pledgeable surplus $\overline{V} - \underline{V}$ was reduced by the introduction of scoring.

First-price auctions and scoring auctions in post period. We now turn to compare first-price auctions and scoring auctions held in the post period. Proposition 4 shows that if either (IC-q) doesn't bind, monitoring is perfect, or transfers are costless, then for auctions held in the post period, the right tail of the distribution of winning bids in scoring auctions should (weakly) first-order stochastically dominate that of first-price auctions. As we will now see, the opposite is true: the c.d.f. of winning bids in first-price auctions was larger (in FOSD terms) than that of scoring auctions. The implication is that monitoring was imperfect and that quality incentive constraints were binding.

Figure 5 is a plot of the c.d.f. of normalized winning bids for first-price and scoring

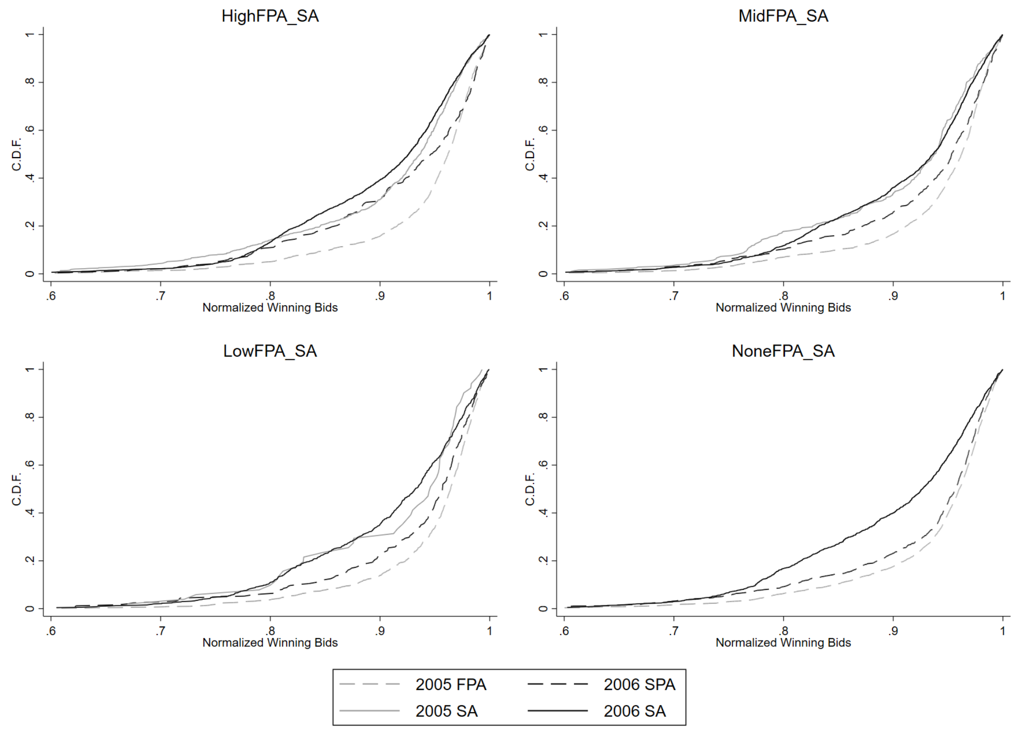

Figure 5: Distribution of winning bids for first-price and scoring auctions. Top left panel corresponds to firms with high exposure to scoring auctions in 2005. Top right and bottom left panel correspond to those with moderate and low exposure, respectively. The bottom right panel corresponds to those with no exposure.

auctions held in 2005 and 2006. The four panels of Figure 5 correspond the subsample of firms with different exposure to scoring auctions in 2005 already used in Figure 4. Each year, the distributions of normalized winning bids in scoring auctions (solid lines) lie to the left of the distribution of normalized winning bids for first-price auctions: it is more difficult to sustain high prices under scoring auctions than under first-price auctions. The implication is that the conditions under which Proposition 4 holds are not verified: (IC-q) is binding, quality is imperfectly monitored, and transfers are costly. Said differently, imperfect monitoring frictions due to the introduction of scoring further limited cartels' ability to operate.

Further evaluation of the imperfect monitoring channel. In order to test further predictions of our model (formalized in Proposition 5), as well as quantify the impact of

	(1)	(2)	(3)	(4)
Scoring	-0.0219***	-0.0232***	-0.0201***	-0.0202***
	(0.0011)	(0.0012)	(0.0014)	(0.0014)
Scoring$\times(\sigma - \overline{\sigma})$		-0.0061***		-0.0062***
		(0.0003)		(0.0003)
Post			-0.0109***	-0.0148***
			(0.0026)	(0.0026)
N	94,050	86,858	55,167	51,534

*, **, and *** respectively denote significance at the 5%, 1%, and 0.1% levels.

Table 5: Regression of Winning Bid: Scoring Auctions vs. First Price Auctions

different frictions on prices, we estimate a linear regression model of the following form

$$winning\_bid_{i,t} = \beta_s scoring_t + \beta_\sigma scoring_t \times (\sigma_t - \bar{\sigma}) + \beta_p \mathsf{post}_{i,t} + \beta X_t + \delta_i + \epsilon_{i,t}, \qquad (2)$$

where the outcome variable is the winning bid as a fraction of the reserve price and *scoring* is a dummy variable taking a value of 1 if the auction is a scoring auction. The interaction between $scoring_t$ and $\sigma_t - \overline{\sigma}$ – the normalized standard error of the quality component of scoring auctions computed at the project category and region level lets us evaluate the effect of an increase in the randomness of the evaluation of the proposals.[18] Higher values of σ implies more randomness and uncertainty in the evaluation of the proposals, and scoring auctions should have a diminished effect if scores are predictable. The variable $\bar{\sigma}$ is the mean of σ in all scoring auctions so the average effect of scoring auctions on winning bid loads on β_s.

The variable post is the measure of exposure to scoring auctions defined earlier. We include project category dummies and full interaction of year and month dummies as controls. Finally, δ_i is a firm fixed effect.

[18]Specifically, for every project category and region combination, we run separate regressions in which we regress the proposal quality of firm i in auction a, $q_{i,a}$, on year dummies and firm fixed effects. We use all of the bids including those of losing bidders to compute σ. The variable σ is the standard error of the residuals.

Table 5 reports our results. Column (1) estimates (2) with only *scoring* as the main regressor. Column (2) includes the interaction between *scoring* and $\sigma - \overline{\sigma}$, and Column (3) includes *scoring* and post. We use the definition of post with 180 days and at least 5 auctions. Column (4) reports the estimates of (2) that includes all three regressors.

Across different specifications, we find that winning bids are about 2 percentage points lower in scoring auctions than in first price auctions let at the same time. We also find that higher uncertainty in the evaluation of proposals leads to a larger decrease in the winning bids. For example, project categories such as prestressed concrete have values of σ above 9. Other categories such as painting and grouting have values of σ less than 3. There are significant regional differences as well: For example, σ is less than 2 in Hokkaido on average, while it is more than 8 in Hokuriku. An increase in the value of σ of 6 is associated with about a 3.6 percentage point (-0.006$\times$ 6) decrease in the winning bid.

This yields three takeaways: scoring auctions reduce winning bids by roughly 1.5% across the board, with an additional 2% reduction for scoring auctions. Within scoring auctions, variation in the uncertainty in quality evaluation mediates the impact of scoring on prices. The impact of scoring is small for auctions with little uncertainty in quality, but roughly doubles for auctions with relatively more random quality evaluations.

8 Conclusion

In competitive environments the introduction of scoring in auctions tends to increase bids. We show theoretically and empirically that when firms collude, the introduction of scoring can lead to significantly lower winning bids. We argue that this outcome is driven by two frictions limiting cartel discipline: imperfect monitoring makes makes incentive provision more expensive in scoring auctions; reduced rents from scoring auctions shrink the pledgeable surplus available to incentivize cartel members even in first-price auctions.

This finding has implications for the way procurement agencies respond to suspected collusion. First, instead of increasing prices, scoring can reduce prices, when collusion among bidders is the main driver of high prices. Furthermore, when collusion is suspected, introducing a qualitative scoring dimension may be an institutionally feasible, and economically effective way to restrain collusion. Proposals should not be disclosed, and quality assessments should be subjective without being excessively noisy. An important caveat to this conclusion is that our data comes from a high governance context, in which careful steps were taken to ensure that the scoring process was not captured by bidders. Introducing subjective scoring in a weak governance context, where corruption rather than collusion is the main driver of high prices, may have opposite effects.

Appendix

Appendix A illustrates the different channels through which scoring can affect collusive behavior in a simplified fully solved model. Appendix B collects proofs for results contained in the paper. Appendix C further substantiates our inference that MLIT procurement auctions were collusive, and provides a more detailed description on how scoring affected bidding behavior.

A An Example

We now illustrate the impact of scoring on cartel discipline with a simple example. The example highlights the role of imperfect monitoring in limiting a cartel's ability to sustain high prices.

Suppose there are two firms that share the same procurement costs: i.e., $\theta_1 = \theta_2 = \theta$. Firms can produce two possible qualities, $Q = \{\underline{q}, \overline{q}\}$. The set of possible recorded qualities is also $\{\underline{q}, \overline{q}\}$. Assume that

$$\text{prob}(\widehat{q}_i = \overline{q} | q_i = \overline{q}) = \text{prob}(\widehat{q}_i = \underline{q} | q_i = \underline{q}) = \alpha \in (1/2, 1).$$

Parameter $\alpha \in (1/2, 1)$ measures how noisy monitoring is under scoring. As α approaches 1 (resp., approaches 1/2), recorded quality becomes a perfect signal (resp., a fully uninformative signal) of a firm's intended quality.

Consider first auction format FPA. Let $b_{\text{FPA}}(\theta)$ denote the largest winning bid that firms can sustain under FPA. Then, for $i = 1, 2$, we must have

$$(1 - x_i)(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta)) \leq \frac{\delta}{2}(\overline{V} - \underline{V}) + T_i(1 + \lambda \mathbf{1}_{T_i<0}),$$

where $x_i \in [0, 1]$ is the probability with which i wins the auction. Indeed, if the above inequality does not hold, firm i has an incentive to undercut the winning bid. Summing this constraint across $i = 1, 2$, and using $x_1 + x_2 \leq 1$ and $T_1 + T_2 \leq 0$ we get that

$$b_{\text{FPA}}(\theta) - c(\underline{q}, \theta) \leq \delta(\overline{V} - \underline{V}). \tag{3}$$

Hence, $b_{\text{FPA}}(\theta) \leq \min\{r, c(\underline{q}, \theta) + \delta(\overline{V} - \underline{V})\}$. Moreover, a winning bid of $\min\{r, c(\underline{q}, \theta) + \delta(\overline{V} - \underline{V})\}$ is enforceable under FPA, by a bidding scheme under which both firms submit bid $\min\{r, c(\underline{q}, \theta) + \delta(\overline{V} - \underline{V})\}$ and quality $\underline{q}$, and each firm wins with probability 1/2. Therefore, $b_{\text{FPA}}(\theta) = \min\{r, c(\underline{q}, \theta) + \delta(\overline{V} - \underline{V})\}$.

Consider next bidding under SA. Let us focus on bidding schemes under which both firms submit the same bid $b_{\text{SA}}(\theta)$ and quality $\underline{q}$. For this bidding profile to be enforceable under SA, there must exist transfers $(T_1(\widehat{q}), T_2(\widehat{q}))_{\widehat{q} \in Q^2}$, with $T_i(\widehat{q}) \geq \underline{T} = -\frac{1}{1+\lambda}\frac{\delta}{2}(\overline{V} - \underline{V})$ for all $\widehat{q}$, such that (IC-q) holds.

The optimal transfer scheme to sustain the proposed bidding profile takes the following intuitive form. When the recorded qualities of both firms are the same, there are no transfers: i.e., $T_i(\widehat{q}, \widehat{q}) = 0$ for all $\widehat{q}$. When firm i's recorded quality is $\underline{q}$ and firm $-i$'s recorded quality is $\overline{q}$, firm i obtains a transfer $T \geq 0$, which is paid by firm $-i$.

Given this bidding and transfer profile, (IC-q) becomes

$$\alpha(b_{\text{SA}}(\theta) - c(\overline{q}, \theta)) - \frac{1}{2}(b_{\text{SA}}(\theta) - c(\underline{q}, \theta)) \leq (2\alpha - 1)((1 - \alpha)T - \alpha T(1 + \lambda \mathbf{1}_{T<0}))$$

The left-hand side of the inequality above is the payoff gain that firm i obtains from submitting quality $\overline{q}$ instead of quality $\underline{q}$: by submitting quality $\overline{q}$, firm i wins with probability $\alpha > 1/2$ instead of 1/2. The right-hand side corresponds to the change in firm i's expected transfers following the deviation. Indeed, $-(2\alpha - 1)(1 - \alpha)$ is the change in the probability that recorded qualities are $(\widehat{q}_i, \widehat{q}_{-i}) = (\underline{q}, \overline{q})$ when i deviates to $q_i = \overline{q}$, and $(2\alpha - 1)\alpha$ is the change in the probability that recorded qualities are $(\widehat{q}_i, \widehat{q}_{-i}) = (\overline{q}, \underline{q})$ following this same deviation.

Using $-T \geq \underline{T} = -\frac{1}{1+\lambda}\frac{\delta}{2}(\overline{V} - \underline{V})$ in the inequality above we get

$$b_{\text{SA}}(\theta) - c(\underline{q}, \theta) \leq \frac{1+\lambda\alpha}{1+\lambda}\delta(\overline{V} - \underline{V}) + \frac{2\alpha}{2\alpha - 1}(c(\overline{q}, \theta) - c(\underline{q}, \theta)). \tag{4}$$

Comparing (4) with (3), the winning bid under SA is lower than under FPA if

$$\begin{aligned} \frac{1+\lambda\alpha}{1+\lambda}\delta(\overline{V} - \underline{V}) + \frac{2\alpha}{2\alpha - 1}(c(\overline{q}, \theta) - c(\underline{q}, \theta)) < \delta(\overline{V} - \underline{V}) \\ \iff \frac{2\alpha}{2\alpha - 1}(c(\overline{q}, \theta) - c(\underline{q}, \theta)) < \frac{\lambda(1-\alpha)}{1+\lambda}\delta(\overline{V} - \underline{V}) \end{aligned}$$

We note that the above inequality is more likely to hold when λ is large (i.e., transfers are costly) and α is neither close to 1/2 nor close to 1 (i.e., monitoring is imperfect but not too much so).

Lastly, when the inequality above holds, and

$$\delta(\overline{V} - \underline{V}) \geq r - c(\underline{q}, \theta)) > \frac{1+\lambda\alpha}{1+\lambda}\delta(\overline{V} - \underline{V}) + \frac{2\alpha}{2\alpha - 1}(c(\overline{q}, \theta) - c(\underline{q}, \theta)),$$

the cartel's payoffs are strictly lower under SA than under FPA. Indeed, when these inequalities hold, the cartel cannot sustain winning bid of r and quality $\underline{q}$ under SA, but can do so under FPA.

B Proofs

Proof of Proposition 1. Consider auction format FPA. Note that, since quality does not affect the allocation, in any Nash equilibrium in weakly undominated strategies all firms choose the lowest quality $q = \underline{q}$. In addition, the bidder with the lowest cost wins with probability 1, and the winning bid is equal to the second lowest cost. Hence, for any type vector $\theta = (\theta_i)_{i \in I}$, the winning bid under FPA is $b^{(1)}_{\text{FPA}}(\theta) = c(\underline{q}, \theta_{(2)})$, where $\theta_{(2)}$ is the second lowest type in θ.

Consider next auction format SA. For each score $s > 0$ and each type θ_i, let $k(\theta_i, s) \equiv \max_{q \in Q} \frac{q}{s} - c(q, \theta_i)$, and let $q^*(\theta_i, s) \in \arg\max_{q \in Q} \frac{q}{s} - c(q, \theta_i)$. Note that $k(\theta_i, s)$ is the largest payoff that firm i with type θ_i can obtain by winning an auction with a score of s. Note further that $k(\theta_i, s)$ is continuous in θ_i and s, and is decreasing in θ_i and in s. Moreover, since, for all $q' > q$, $c(q', \theta_i) - c(q, \theta_i)$ is strictly increasing in θ_i, it follows that $q^*(\theta'_i, s) \leq q^*(\theta_i, s)$

for all $s > 0$ and all $\theta_i < \theta_i'$.[19]

Then, in any equilibrium in weakly undominated strategies, for any type vector $\theta = (\theta_i)_{i \in I}$, the bidder with the lowest type $\theta_{(1)}$ wins, and the winning score $s_{\text{SA}}^{\text{comp}}(\theta)$ is such that the bidder with the second lowest type would make zero profits winning with that score: $s_{\text{SA}}^{\text{comp}}(\theta) = \inf\{s : k(\theta_{(2)}, s) \leq 0\}$.[20] The winning bid $b_{\text{SA}}^{\text{comp}}(\theta)$ and winning quality $q_{\text{SA}}^{\text{comp}}(\theta)$ satisfy $b_{\text{SA}}^{\text{comp}}(\theta) = \frac{q_{\text{SA}}^{\text{comp}}(\theta)}{s_{\text{SA}}^{\text{comp}}(\theta)}$, and the winning quality is optimal to the winning firm; i.e., $q_{\text{SA}}^{\text{comp}}(\theta) = q^*(\theta_{(1)}, s_{\text{SA}}^{\text{comp}}(\theta)) \in \arg\max_q \frac{q}{s_{\text{SA}}^{\text{comp}}(\theta)} - c(q, \theta_{(1)})$.

Lastly, we show that, for all θ, $b_{\text{SA}}^{\text{comp}}(\theta) \geq b_{\text{FPA}}^{\text{comp}}(\theta)$. The winning score $s_{\text{SA}}^{\text{comp}}(\theta) = q_{\text{SA}}^{\text{comp}}(\theta)/b_{\text{SA}}^{\text{comp}}(\theta)$ is such that $\forall q \in [\underline{q}, \overline{q}]$,

$$\frac{q}{s_{\text{SA}}^{\text{comp}}(\theta)} - c(q, \theta_{(2)}) \leq 0,$$

with equality at $q = q^*(\theta_{(2)}, s_{\text{SA}}^{\text{comp}}(\theta)) \in \arg\max_q \frac{q}{s_{\text{SA}}^{\text{comp}}(\theta)} - c(q, \theta_{(2)})$. We then have that,

$$\begin{aligned} b_{\text{SA}}^{\text{comp}}(\theta) &= \frac{q_{\text{SA}}^{\text{comp}}(\theta)}{s_{\text{SA}}^{\text{comp}}(\theta)} \\ &= \frac{c(q^*(\theta_{(2)}, s_{\text{SA}}^{\text{comp}}(\theta)), \theta_{(2)})}{q^*(\theta_{(2)}, s_{\text{SA}}^{\text{comp}}(\theta))} q_{\text{SA}}^{\text{comp}}(\theta) \geq c(\underline{q}, \theta_{(2)}) = b_{\text{FPA}}^{\text{comp}}(\theta), \end{aligned}$$

where the inequality follows since $q_{\text{SA}}^{\text{comp}}(\theta) = q^*(\theta_{(1)}, s_{\text{SA}}^{\text{comp}}(\theta)) \geq q^*(\theta_{(2)}, s_{\text{SA}}^{\text{comp}}(\theta))$, and since $q^*(\theta_{(2)}, s_{\text{SA}}^{\text{comp}}(\theta)) \geq \underline{q}$. ■

Proof of Proposition 2. Consider a profile of types θ. For simplicity, index by 1 the bidder with the lowest cost $c(\underline{q}, \theta_1)$ at type profile θ. In a weakly undominated Nash equilibrium of the FPA, bidder 1 sets $b_{1,FPA} = \min\{c(\underline{q}, \theta_j), j \in \{2, \cdots, n\}\}$, while other

[19]Proof: Pick $\theta_i' > \theta_i$. Note that, for any $s > 0$, $\frac{q^*(\theta_i, s)}{s} - c(q^*(\theta_i, s), \theta_i) \geq \frac{q^*(\theta_i', s)}{s} - c(q^*(\theta_i', s), \theta_i)$ and $\frac{q^*(\theta_i', s)}{s} - c(q^*(\theta_i', s), \theta_i') \geq \frac{q^*(\theta_i, s)}{s} - c(q^*(\theta_i, s), \theta_i')$. These two inequalities imply

$$c(q^*(\theta_i', s), \theta_i) - c(q^*(\theta_i, s), \theta_i) \geq c(q^*(\theta_i', s), \theta_i') - c(q^*(\theta_i, s), \theta_i').$$

Since, for all $q' > q$, $c(q', \hat{\theta}_i) - c(q, \hat{\theta}_i)$ is strictly increasing in $\hat{\theta}_i$, it follows that $q^*(\theta_i', s) \leq q^*(\theta_i, s)$.

[20]To see why, note that in any equilibrium in weakly undominated strategies, bidders with type $\theta_i \geq \theta_{(2)}$ must submit a price-quality bid (b, q) with score weakly smaller than $s_{\text{SA}}^{\text{comp}}(\theta)$. Indeed, submitting a price-quality bid (b, q) with a score strictly larger than $s_{\text{SA}}^{\text{comp}}(\theta)$ is dominated for bidders with type $\theta_i \geq \theta_{(2)}$, as they would lose money by winning the auction at such a score. This implies that the bidder with the highest type $\theta_{(1)}$ must win the auction with probability 1: otherwise, it would be a strictly profitable deviation for this bidder to 'overcut' the highest score among its rivals and win the auction with probability 1. Note next that the highest type's score must be weakly smaller than $s_{\text{SA}}^{\text{comp}}(\theta)$, since all other bidders submit scores weakly smaller than this number. Lastly, if the winning score was strictly smaller than $s_{\text{SA}}^{\text{comp}}(\theta)$, the bidder with the second highest type $\theta_{(2)}$ would find it strictly profitable to overcut the winning score.

bidders $i \neq 1$ set $b_{i,FPA} = c(\underline{q}, \theta_i)$. This yields that $b^{(2)}_{FPA} = \min\{c(\underline{q}, \theta_i)\}$.

Quality can only increase in a weakly undominated Nash equilibrium of the FPA. As a result it must be that for all $i \neq 1$ bids mechanically increase under the SA: $b_{i,SA} \geq c(\underline{q}, \theta_i)$.

In addition, it must be that $b^{(2)}_{SA} \geq \min\{b_{i,SA}, j \in \{2, \cdots, n\}\}$. Hence, it follows that $b^{(2)}_{SA} \geq b^{(2)}_{FPA}$. ■

Proof of Lemma 1. Fix a type profile θ. Then, for any any bidding profile $b = (b_i)_{i \in I}, q = (q_i)_{i \in I}$ and on-path transfer profile $T : Q^I \to \mathbb{R}^n$, the cartel's flow payoff is

$$\sum_i u_i(b, q, \theta) - \lambda \mathbb{E}_\mu \left[\sum_i [T_i(\widehat{q})]^- \right]. \tag{5}$$

For the bidding and transfer profile to be sustainable in a Stationary PPE, the following constraints must hold:

$$\forall i \in I, \forall b'_i \neq b_i, \forall q'_i, \quad u_i(b'_i, b_{-i}, q'_i, q_{-i}, \theta) - u_i(b, q, \theta) \tag{6}$$
$$\leq \sum_{\widehat{q} \in Q^I} T_i(\widehat{q})(1 + \lambda \mathbf{1}_{T_i(\widehat{q})<0}) \mu(\widehat{q}|q) + \frac{\delta}{n}(\overline{V} - \underline{V})$$
$$\forall i \in I, \forall q'_i \neq q_i, \quad u_i(b, q'_i, q_{-i}, \theta) - u_i(b, q, \theta) \tag{7}$$
$$\leq \sum_{\widehat{q} \in Q^I} T_i(\widehat{q})(1 + \lambda \mathbf{1}_{T_i(\widehat{q})<0})(\mu(\widehat{q}|q) - \mu(\widehat{q}|q'_i, q_{-i}))$$
$$\forall i \in I, \forall \widehat{q} \in Q^I, -T_i(\widehat{q})(1 + \lambda \mathbf{1}_{T_i(\widehat{q})<0}) \leq \frac{\delta}{n}(\overline{V} - \underline{V}) \tag{8}$$
$$\forall \widehat{q} \in Q^I, \sum_i T_i(\widehat{q}) = 0. \tag{9}$$

Constraint (6) states that no bidder i can gain by defecting and placing a bid $b'_i \neq b_i$: since such deviations are detectable, they are punished with Nash reversion. Constraint (7) states that no bidder i can gain by defecting and placing a bid with intended quality $q'_i \neq q_i$ (without changing the bid b_i): since such deviations are not detectable (because $\mu(\cdot|q)$ has full support), and since the equilibrium is on-path stationary, such deviations can only be deterred using transfers on the equilibrium path. Constraint (8) guarantees that bidders have an incentive to pay their corresponding transfers (with punishment for failing to pay a transfer taking the form of Nash reversion). Lastly, constraint (9) says that transfers must be budget-balance.

The cartel-optimal bidding and transfer profile b, q, T when bidders' types are θ maximizes

(5) subject to (6), (7), (8) and (9). This program can be decomposed as follows. For each b, q, find transfers $T : Q^I \to [\underline{T}, \infty)^I$ (with $\underline{T} \equiv -\frac{1}{1+\lambda}\frac{\delta}{n}(\overline{V} - \underline{V})$) that solve

$$K(b, q, \theta) = \min_{T:Q^I \to [\underline{T},\infty)^I} \mathbb{E}_\mu \left[\sum_i [T_i(\widehat{q})]^- \right],$$

subject to (6), (7) and (9).[21] Then, the cartel's problem is to find b, q that maximize $\sum_i u_i(b, q, \theta) - \lambda K(b, q, \theta)$. This completes the proof. ■

Proof of Proposition 3. Follows from the characterization of optimal bidding behavior in Lemma 1, and from the fact that (IC-q) is not relevant under FPA. ■

For any vector of types $\theta = (\theta_i)_{i \in I}$, we let $\underline{c}(\theta) = \min_i c(\underline{q}, \theta_i)$ and $\overline{c}(\theta) = \max_i c(\underline{q}, \theta_i)$. We also let $b_{\text{FPA}}(\theta)$ denote the winning bid under that solves program (P) under FPA when firms' types are θ.

Lemma B.1. *Suppose $\delta(\overline{V} - \underline{V}) > 0$, and let θ be such that $\max_{i,j} |c(\underline{q}, \theta_i) - c(\underline{q}, \theta_j)| < \eta$ for $\eta \in (0, \frac{\delta}{n-1}(\overline{V} - \underline{V}))$. Then, $b_{FPA}(\theta) \geq \min\{r, \underline{c}(\theta) + \frac{\delta}{n-1}(\overline{V} - \underline{V}) - \eta\}$.*

Proof. Suppose not, so that $b_{\text{FPA}}(\theta) < \underline{c}(\theta) + \frac{\delta}{n-1}(\overline{V} - \underline{V}) - \eta$. Note then that cartel flow profits (including transfers) when types are θ are bounded above by

$$b_{\text{FPA}}(\theta) - \underline{c}(\theta) < \frac{\delta}{n-1}(\overline{V} - \underline{V}) - \eta.$$

Let $\hat{b} = \underline{c}(\theta) + \frac{\delta}{n-1}(\overline{V} - \underline{V})$, and consider the following bidding and transfer profile: all firms bid $\hat{b}$ and quality $\underline{q}$, and there are no transfers. Note that this bidding profile is enforceable under FPA when firms' types are θ. Indeed, each i gets a payoff of $\frac{1}{n}(\hat{b} - c(\underline{q}, \theta_i)) + \frac{\delta}{n}\overline{V}$ by following this strategy profile, and gets $\hat{b} - c(\underline{q}, \theta_i) + \frac{\delta}{n}\underline{V}$ by undercutting bid $\hat{b}$. For each $i \in I$, the deviation is not profitable if and only if

$$\hat{b} \leq c(\underline{q}, \theta_i) + \frac{\delta}{n-1}(\overline{V} - \underline{V}),$$

which holds since $\underline{c}(\theta) = \min_i c(\underline{q}, \theta_i)$. Hence, this bidding and transfer profile is enforceable when bidders' types are θ. Moreover, note that cartel flow profits under this profile are

[21] By convention, we set $K(b, q, \theta) = +\infty$ if b, q are such that there are no transfers $T : Q^I \to [\underline{T}, \infty)^I$ satisfying (6), (7) and (9).

weakly larger than

$$\hat{b} - \overline{c}(\theta) = \frac{\delta}{n-1}(\overline{V} - \underline{V}) - (\overline{c}(\theta) - \underline{c}(\theta)) \geq \frac{\delta}{n-1}(\overline{V} - \underline{V}) - \eta.$$

Hence, this bidding and transfer profile leads to strictly larger cartel profits than the optimal one (with a winning bid $b_{\mathrm{FPA}}(\theta) < \underline{c}(\theta) + \frac{\delta}{n-1}(\overline{V} - \underline{V}) - \eta$), a contradiction. ■

Proof of Proposition 4. Fix a profile of types θ such that $\max_{i,j} |c(\underline{q}, \theta_i) - c(\underline{q}, \theta_j)| < \eta$ for some $\eta > 0$ to be determined shortly. Recall that $b_{\mathrm{FPA}}(\theta)$ is the winning bid that solves program (P) under FPA when firms' types are θ, and that $\overline{c}(\theta) = \max_i c(\underline{q}, \theta_i)$ and $\underline{c}(\theta) = \min_i c(\underline{q}, \theta_i)$. Note that $b_{\mathrm{FPA}}(\theta)$ must be such that, for each $i \in I$,

$$(1 - x_i)(b_{\mathrm{FPA}}(\theta) - c(\underline{q}, \theta_i)) \leq \frac{\delta}{n}(\overline{V} - \underline{V}) + T_i(1 + \lambda \mathbf{1}_{T_i<0}),$$

where $x_i \in [0, 1]$ is the probability that i wins the auction, and T_i is i's net transfer. Summing this inequality over all i, and using $\sum_i x_i \leq 1$, $\sum_i T_i = 0$ and $c(\underline{q}, \theta_i) \leq \overline{c}(\theta)$ for all i, we get

$$\frac{n-1}{n}(b_{\mathrm{FPA}}(\theta) - \overline{c}(\theta)) \leq \frac{\delta}{n}(\overline{V} - \underline{V}) \Longleftrightarrow b_{\mathrm{FPA}}(\theta) \leq \overline{c}(\theta) + \frac{\delta}{n-1}(\overline{V} - \underline{V}). \tag{10}$$

Consider the following bidding and transfer profile under SA: all bidders place bid $b_{\mathrm{FPA}}(\theta)$ and intended quality $\underline{q}$, and there are no transfers (i.e., $T_i(\widehat{q}) = 0$ for all $\widehat{q}$ and all i). Clearly, these transfers are feasible (i.e., for all $\widehat{q}$, $\sum_i T_i(\widehat{q}) = 0$, and, for all i, $T_i(\widehat{q}) \geq \underline{T}$).

Let $(b, q) = (b_i, q_i)_{i \in I}$ be the bidding profile under which all bidders submit bid $b_{\mathrm{FPA}}(\theta)$ and intended quality $\underline{q}$. Bidder i's payoff under SA under this bidding profile is $u_i(b, q, \theta_i) = \frac{1}{n}(b_{\mathrm{FPA}}(\theta) - c(\underline{q}, \theta_i))$. Note that there exists $\alpha_1 \in (1/n, 1)$ independent of $b_{\mathrm{FPA}}(\theta)$ such that, for all $b_i' < b_{\mathrm{FPA}}(\theta)$ and all q_i', $u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) \le \alpha_1(b_{\mathrm{FPA}}(\theta) - c(\underline{q}, \theta_i))$.[22] Hence,

$$\begin{aligned}\forall i, \forall b_i' < b_{\mathrm{FPA}}(\theta), \forall q_i', \quad u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) - u_i(b, q, \theta_i) &\le \left(\alpha_1 - \frac{1}{n}\right)(b_{\mathrm{FPA}}(\theta) - c(\underline{q}, \theta_i)) \\ &\le \left(\alpha_1 - \frac{1}{n}\right)(b_{\mathrm{FPA}}(\theta) - \underline{c}(\theta)).\end{aligned} \tag{11}$$

Define $\eta_1 \equiv \delta(\overline{V} - \underline{V})\left(\frac{1}{n\alpha_1 - 1} - \frac{1}{n-1}\right)$, and let $\eta \in (0, \eta_1)$. Note then that

$$\begin{aligned}\left(\alpha_1 - \frac{1}{n}\right)(b_{\mathrm{FPA}}(\theta) - \underline{c}(\theta)) &\le \left(\alpha_1 - \frac{1}{n}\right)\left(\overline{c}(\theta) - \underline{c}(\theta) + \frac{\delta}{n-1}(\overline{V} - \underline{V})\right) \\ &\le \left(\alpha_1 - \frac{1}{n}\right)\left(\eta + \frac{\delta}{n-1}(\overline{V} - \underline{V})\right) \le \frac{\delta}{n}(\overline{V} - \underline{V}),\end{aligned}$$

where the first inequality uses (10), the second inequality uses $\overline{c}(\theta) - \underline{c}(\theta) \le \eta$ and the last inequality uses $\eta \le \eta_1$. Combining this with (11), we get

$$\forall i, \forall b_i' < b_{\mathrm{FPA}}(\theta), \forall q_i' \quad u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) - u_i(b, q, \theta_i) \le \frac{\delta}{n}(\overline{V} - \underline{V}). \tag{12}$$

There are two cases to consider: (a) $b_{\mathrm{FPA}}(\theta) = r$, and (b) $b_{\mathrm{FPA}}(\theta) < r$. In case (a), the inequalities in (12) imply that the proposed bidding and transfer profile satisfy (IC-p) under auction format SA.

Consider next case (b). By Lemma B.1, we have that

$$r > b_{\mathrm{FPA}}(\theta) \ge \underline{c}(\theta) + \frac{\delta}{n-1}(\overline{V} - \underline{V}) - \eta \Longrightarrow \overline{c}(\theta) < \underline{c}(\theta) + \eta < r - \frac{\delta}{n-1}(\overline{V} - \underline{V}) + 2\eta. \tag{13}$$

Since $\gamma(\widehat{q}_i|q_i')$ has full support over Q for all q_i', there exists $\alpha_2 \in (1/n, 1)$ such that, for all $q_i' \neq \underline{q}$ and all $b_i' > b_{\mathrm{FPA}}(\theta)$, $D_i(b_i', b_{-i}, q_i', q_{-i}) \le \alpha_2$. Hence, for all $i \in I$, all $b_i' > b_{\mathrm{FPA}}(\theta)$ and all q_i', $u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) \le \alpha_2(r - c(\underline{q}, \theta_i))$. Define

$$\eta_2 \equiv \frac{1 - \alpha_2}{2n(1 - \alpha_2) + n - 1}\frac{n}{n-1}\delta(\overline{V} - \underline{V}),$$

[22] To see why the claim is true, note that if $b_i' \in (b_{\mathrm{FPA}}(\theta)\underline{q}/\overline{q}, b_{\mathrm{FPA}}(\theta))$, then by the full support assumption the probability that i wins the auction by bidding b_i', q_i' when all others bid $b_{\mathrm{FPA}}(\theta), \underline{q}$ is bounded by some $\alpha < 1$. And so the payoff i gets from bidding the deviation is bounded by $\alpha(b_{\mathrm{FPA}}(\theta) - c(\underline{q}, \theta_i))$. On the other hand, the payoff that i obtains by bidding $b_i' \le b_{\mathrm{FPA}}(\theta)\underline{q}/\overline{q}$ is bounded by $b_{\mathrm{FPA}}(\theta)(\underline{q}/\overline{q}) - c(\underline{q}, \theta_i) < (\underline{q}/\overline{q})(b_{\mathrm{FPA}}(\theta) - c(\underline{q}, \theta_i))$. Letting $\alpha_1 = \max\{\alpha, \underline{q}/\overline{q}\}$ establishes the claim.

and assume that $\eta \in (0, \eta_2)$. Define

$$\underline{b} \equiv r - (1-\alpha_2)\frac{\delta}{n-1}(\overline{V} - \underline{V}) + \eta\left(2(1-\alpha_2) + \frac{n-1}{n}\right).$$

Note that $\eta < \eta_2$ implies $\underline{b} < r$. Suppose $b_{\text{FPA}}(\theta) \geq \underline{b}$. Then,

$$\begin{aligned}\forall i, \forall b_i' > b_{\text{FPA}}(\theta), \forall q_i', \quad u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) - u_i(b, q, \theta_i) &\leq \alpha_2(r - c(\underline{q}, \theta_i)) - \frac{1}{n}(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i)) \\ &\leq \frac{n-1}{n}(b_{\text{FPA}}(\theta) - \overline{c}(\theta)) \leq \frac{\delta}{n}(\overline{V} - \underline{V}).\end{aligned} \tag{14}$$

The first inequality follows from the bound on $u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i)$ derived above, and the last inequality follows from (10). To see that the middle inequality holds, note that

$$\begin{aligned}\alpha_2(r - c(\underline{q}, \theta_i)) - \frac{1}{n}(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i)) &\leq \frac{n-1}{n}(b_{\text{FPA}}(\theta) - \overline{c}(\theta)) \\ \iff b_{\text{FPA}}(\theta) &\geq \alpha_2 r + (1-\alpha_2)c(\underline{q}, \theta_i) + \frac{n-1}{n}(\overline{c}(\theta) - c(\underline{q}, \theta_i)),\end{aligned}$$

which is always satisfied since $b_{\text{FPA}}(\theta) \geq \underline{b}$, $\overline{c}(\theta) - c(\underline{q}, \theta_i) \leq \eta$ and since, by (13), $c(\underline{q}, \theta_i) \leq r - \frac{\delta}{n-1}(\overline{V} - \underline{V}) + 2\eta$. Hence, if $\eta < \min\{\eta_1, \eta_2\}$, and if $b_{\text{FPA}}(\theta) \geq \underline{b}$, then (14) and (12) both hold, and so the proposed bidding and transfer scheme satisfies (IC-b).

Therefore, if $\max_{i,j} |c(\underline{q}, \theta_i) - c(\underline{q}, \theta_j)| < \eta$ for $\eta < \min\{\eta_1, \eta_2\}$, and if either (IC-q) is not binding, or if quality is noisily evaluated but perfectly monitored, then enforceability of $b \geq \underline{b}$ under FPA implies enforceability of b under SA.[23]

Lastly, we consider the case in which transfers are costless, so that $\lambda = 0$. Fix a vector of types θ such that $\max_{i,j} |c(\underline{q}, \theta_i) - c(\underline{q}, \theta_j)| < \eta$, and let again $b_{\text{FPA}}(\theta)$ be the optimal winning bid under FPA. By the arguments above, $b_{\text{FPA}}(\theta)$ satisfies (10). Consider the following bidding and transfer profile under SA when bidders' types are θ. Each bidder i submits bid $b_i = b_{\text{FPA}}(\theta)$ and intended quality $q_i = \underline{q}$. If bidder i wins the auction, it pays transfer $T_i = -\frac{n-1}{n}(b_{\text{FPA}}(\theta) - \overline{c}(\theta))$, which is divided evenly among all losing bidders; i.e., each loser $j \neq i$ gets $T_j = -\frac{1}{n-1}T_i = \frac{1}{n}(b_{\text{FPA}}(\theta) - \overline{c})$. Since $b_{\text{FPA}}(\theta)$ satisfies (10), it follows that $T_j \geq \underline{T}$ for all i, so the transfer profile is feasible.

We now show that, if $\eta > 0$ is small and if $b_{\text{FPA}}(\theta) \geq \underline{b}$ (with $\underline{b} < r$ defined above), this

[23]If quality is noisily evaluated but perfectly monitored, then the only relevant constraints for enforceability are (IC-p) and the constraints on transfers. Indeed, (IC-q) is not relevant, since quality deviations are detected and can be punished with Nash reversion.

bidding and transfer profile also satisfy (IC-p) and (IC-q), and so it's enforceable. We start by showing that, under these conditions, (IC-p) holds. Let $(b, q) = (b_i, q_i)_{i \in I}$ be such that, for all i, $b_i = b_{\text{FPA}}(\theta)$ and $q_i = \underline{q}$. Note that, for all i, $u_i(b, q, \theta_i) = \frac{1}{n}(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i))$: under this bidding profile, each bidder wins with probability $1/n$, and pays cost $c(\underline{q}, \theta_i)$ when it wins. Note further that $\sum_{\widehat{q}} \mu(\widehat{q}|q) T_i(\widehat{q}) = 0$; i.e., on average, each bidder pays zero transfers.

In addition, and by the same arguments as above, there exists $\alpha_1 < 1$ such that, for all $b_i' < b_{\text{FPA}}(\theta)$ and for all q_i', $u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) \leq \alpha_1(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i))$. Hence, the same arguments we used above imply that, if $\eta > 0$ is smaller than η_1, then (12) holds. In particular, if $b_{\text{FPA}}(\theta) = r$, the (IC-p) holds.

Suppose next that $b_{\text{FPA}}(\theta) < r$. The same arguments used above imply that there exists $\alpha_2 < 1$ such that, for all i, all $b_i' > b_{\text{FPA}}(\theta)$ and all q_i', $u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) \leq \alpha_2(r - c(\underline{q}, \theta_i))$. Hence, the same arguments we used above imply that, if $\eta > 0$ is smaller than η_2, and if $b_{\text{FPA}}(\theta) \geq \underline{b}$, then (14) also holds. Hence, if $\eta < \min\{\eta_1, \eta_2\}$ and $b_{\text{FPA}}(\theta) \geq \underline{b}$, then the proposed bidding and transfer profile satisfies (IC-p).

We now show that there exists $\eta_3 > 0$ such that, if $\eta < \eta_3$, then the proposed bidding and transfer profile also satisfies (IC-q). To see why, note that, for all i and all q_i',

$$
\begin{aligned}
u_i(b, q_i', q_{-i}, \theta_i) + \sum_{\widehat{q}} \mu(\widehat{q}|q_i', q_{-i}) T_i(\widehat{q}) &= D_i(b, q_i', q_{-i})(b_{\text{FPA}}(\theta) - c(q_i', \theta_i)) + \sum_{\widehat{q}} \mu(\widehat{q}|q_i', q_{-i}) T_i(\widehat{q}) \\
&= \text{prob}(i \text{ wins}|q_i', q_{-i}) \left(b_{\text{FPA}}(\theta) - c(q_i', \theta_i) - \frac{n-1}{n}(b_{\text{FPA}}(\theta) - \overline{c}(\theta)) \right) \\
&\quad + \text{prob}(i \text{ loses}|q_i', q_{-i}) \frac{1}{n}(b_{\text{FPA}}(\theta) - \overline{c}(\theta)) \\
&= \frac{1}{n}(b_{\text{FPA}}(\theta) - \overline{c}(\theta)) + \text{prob}(i \text{ wins}|q_i', q_{-i})(\overline{c}(\theta) - c(q_i', \theta_i)).
\end{aligned}
$$

Hence, (IC-q) holds if and only if

$$
\forall i, \forall q_i' \neq \underline{q}, \quad \text{prob}(i \text{ wins}|q_i', q_{-i})(\overline{c}(\theta) - c(q_i', \theta_i)) \leq \text{prob}(i \text{ wins}|q)(\overline{c}(\theta) - c(\underline{q}, \theta_i)),
$$

which holds for all $\eta < \eta_3 \equiv \min_{\theta_i \in [\underline{\theta}, \overline{\theta}], q_i' \in Q \setminus \{\underline{q}\}} c_i(q_i', \theta_i) - c_i(\underline{q}, \theta_i)$.[24] Hence, for $\eta < \min\{\eta_1, \eta_2, \eta_3\}$, the proposed bidding and transfer profile satisfies (IC-q) and (IC-p) whenever $b_{\text{FPA}}(\theta) \geq \underline{b}$.

Therefore, if $\max_{i,j} |c(\underline{q}, \theta_i) - c(\underline{q}, \theta_j)| < \eta$ for $\eta < \min\{\eta_1, \eta_2, \eta_3\}$, and if $\lambda = 0$, then enforceability of $b \geq \underline{b}$ under FPA implies enforceability of b under SA. ■

[24]Since Q is discrete, $c_i(q_i', \theta_i) > c_i(\underline{q}, \theta_i)$ for all $q_i' \in Q \setminus \{\underline{q}\}$ and all θ_i. Since $c(q, \theta_i)$ is continuous in θ_i, we have that $\eta_3 > 0$.

Proof of Proposition 5. Fix $\theta = (\theta_i)_{i \in I}$ such that $\max_{i \neq j} |c(\underline{q}, \theta_i) - c(\underline{q}, \theta_j)| < \eta$, and let $b_{\text{FPA}}(\theta)$ denote the optimal enforceable winning bid under FPA when bidders' types are θ. From the proof of Proposition 4, we know that the following bidding and transfer profile under SA satisfies (IC-p) whenever $\eta < \min\{\eta_1, \eta_2\}$, and $b_{\text{FPA}}(\theta) \geq \underline{b}$: all bidders bid $b_i = b_{\text{FPA}}(\theta)$ and quality $q_i = \underline{q}$, and there are no transfers. We now show that, if $\min_{q' \neq q, \theta_i} |c(q', \theta_i) - c(q, \theta_i)| > 1/\epsilon$ for $\epsilon > 0$ small enough, or if $\sup_{q_i' \neq q_i'', \widehat{q}} |\ln \gamma(\widehat{q}|q_i') - \ln \gamma(\widehat{q}|q_i'')| < \epsilon$ for ϵ small enough, this bidding and transfer profile also satisfies (IC-q). Note that this implies that the bidding and transfer profile with winning bid $b_{\text{FPA}}(\theta)$ is enforceable under SA. Hence, if $\max_{i \neq j} |c(\underline{q}, \theta_i) - c(\underline{q}, \theta_j)| < \eta$, enforceability of $b \geq \underline{b}$ under FPA implies enforceability of b under SA.

Consider first the case in which $\min_{q_i' \neq q_i'', \theta_i'} |c(q_i', \theta_i') - c(q_i'', \theta_i)| > 1/\epsilon$, with $\epsilon \in (0, 1/r)$. Note that this implies that, for all i and all $q_i' \neq \underline{q}$, $c(q_i', \theta_i) > c(\underline{q}, \theta_i) + r \geq r$. Then,

$$\forall i, \forall q_i' \neq \underline{q}, \quad u_i(b, q_i', \theta_i) - u_i(b, q, \theta_i) = D_i(b, q_i', q_{-i})(b_{\text{FPA}}(\theta) - c(q_i', \theta_i)) - \frac{1}{n}(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i)) < 0,$$

where the strict inequality follows since $c(q_i', \theta_i) > r \geq b_{\text{FPA}}(\theta)$. Since there are no transfers, the bidding and transfer profile satisfies (IC-q).

Suppose next that $\sup_{q_i' \neq q_i'', \widehat{q}} |\ln \gamma(\widehat{q}|q_i') - \ln \gamma(\widehat{q}|q_i'')| < \epsilon$ for $\epsilon > 0$ small. Note that there exists $\varepsilon(\epsilon) > 0$, with $\varepsilon(\epsilon) \to 0$ as $\epsilon \to 0$, such that for all $q_i' \neq \underline{q}$, $|D_i(b, q_i', q_{-i}) - D_i(b, q)| = |D_i(b, q_i', q_{-i}) - \frac{1}{n}| \leq \varepsilon(\epsilon)$. Hence,

$$\begin{aligned}\forall i, \forall q_i' \neq \underline{q}, \quad u_i(b, q_i', \theta_i) - u_i(b, q, \theta_i) &\leq (\frac{1}{n} + \varepsilon(\epsilon))(b_{\text{FPA}}(\theta) - c(q_i', \theta_i)) - \frac{1}{n}(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i)) \\ &= \frac{1}{n}(c(\underline{q}, \theta_i) - c(q_i', \theta_i)) + \varepsilon(\epsilon)(b_{\text{FPA}}(\theta) - c(q_i', \theta_i))\end{aligned}$$

Since $c(q_i', \theta_i) - c(\underline{q}, \theta_i) \geq \min_{q_i'' \neq \underline{q}, \theta_i'} c(q_i'', \theta_i') - c(\underline{q}, \theta_i') > 0$, it follows that $u_i(b, q_i', \theta_i) - u_i(b, q, \theta_i) < 0$ for all $\epsilon > 0$ smaller than some $\overline{\epsilon} > 0$. Since the proposed bidding and transfers profile has no transfers, the profile satisfies (IC-q).

Suppose next that $\max_{q' \neq q, \theta_i} |c(q', \theta_i) - c(q, \theta_i)| < \epsilon$ for ϵ small, and consider the following bidding and transfer profile under SA: all bidders bid $b_i = b_{\text{FPA}}(\theta)$ and $q_i = \overline{q}$, and there no transfers. We now show that, when $\eta < 0$ and $\epsilon > 0$ are both small enough, and when $b_{\text{FPA}}(\theta)$ is larger than some $\underline{b}' < r$, this bidding and transfer profile satisfies (IC-p) and (IC-q). Hence, if $\max_{i \neq j} |c(\underline{q}, \theta_i) - c(\underline{q}, \theta_j)| < \eta$, enforceability of $b \geq \underline{b}'$ under FPA implies enforceability of b under SA.

Each bidder's payoff under SA under bidding profile (b, q) is $u_i(b, q, \theta_i) = \frac{1}{n}(b_{\text{FPA}}(\theta) -$

$c(\overline{q}, \theta_i)) \geq \frac{1}{n}(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i)) - \frac{\epsilon}{n}$. By the same arguments as in the proof of Proposition 4, there exists $\alpha_1 \in (1/n, 1)$ independent of $b_{\text{FPA}}(\theta)$ such that, for all $b_i' < b_{\text{FPA}}(\theta)$, and all q_i', $u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) \leq \alpha_1(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i))$. Hence,

$$\begin{aligned}\forall i, \forall b_i' < b_{\text{FPA}}(\theta), \forall q_i' \quad u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) - u_i(b, q, \theta_i) &\leq \left(\alpha_1 - \frac{1}{n}\right)(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i)) + \frac{\epsilon}{n} \\ &\leq \left(\alpha_1 - \frac{1}{n}\right)(b_{\text{FPA}}(\theta) - \underline{c}(\theta)) + \frac{\epsilon}{n} \\ &\leq \left(\alpha_1 - \frac{1}{n}\right)\left(\eta + \frac{\delta}{n-1}(\overline{V} - \underline{V})\right) + \frac{\epsilon}{n},\end{aligned} \tag{15}$$

where the first inequality uses the bound on $u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i)$ derived above, the second one uses $\underline{c}(\theta) \leq c(\underline{q}, \theta_i)$, and the third one uses (10) and $\overline{c}(\theta) - \underline{c}(\theta) \leq \eta$. Note that, for $\eta < \eta_1$ (with η_1 as defined in the proof of Proposition 4) and for ϵ smaller than some $\overline{\epsilon}(\eta) > 0$, the right-hand side of (15) is smaller than $\frac{\delta}{n}\delta(\overline{V} - \underline{V})$. Hence, for η and ϵ small enough,

$$\forall i, \forall b_i' < b_{\text{FPA}}(\theta), \forall q_i' \quad u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) - u_i(b, q, \theta_i) \leq \frac{\delta}{n}(\overline{V} - \underline{V}), \tag{16}$$

If $b_{\text{FPA}}(\theta) = r$, then (16) implies that the bidding and transfer profile satisfies (IC-p).

Suppose next that $b_{\text{FPA}}(\theta) < r$. Note then that, for all $i \in I$, all $b_i' > b_{\text{FPA}}(\theta)$ and all q_i', $u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) \leq \frac{1}{n}(r - c(\underline{q}, \theta_i))$. Indeed, i's probability of winning when bidding $b_i' > b_{\text{FPA}}(\theta)$ and q_i' must be weakly lower than i's probability of winning when bidding $b_i = b_{\text{FPA}}(\theta)$ and $q_i = \overline{q}$. Define $\underline{b}' \equiv r - \delta(\overline{V} - \underline{V}) + \epsilon$ and suppose $b_{\text{FPA}}(\theta) \geq \underline{b}'$. Then,

$$\begin{aligned}\forall i, \forall b_i' > b_{\text{FPA}}(\theta), \forall q_i', \quad u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) - u_i(b, q, \theta_i) &\leq \frac{1}{n}(r - c(\underline{q}, \theta_i)) - \frac{1}{n}(b_{\text{FPA}}(\theta) - c(\overline{q}, \theta_i)) \\ &\leq \frac{1}{n}(r - b_{\text{FPA}}(\theta)) + \frac{1}{n}\epsilon \\ &\leq \frac{\delta}{n}(\overline{V} - \underline{V}),\end{aligned} \tag{17}$$

where the first inequality uses $u_i(b, q, \theta_i) = \frac{1}{n}(b_{\text{FPA}}(\theta) - c(\overline{q}, \theta_i))$ and the bound on $u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i)$ derived above, the second inequality uses $c(\underline{q}, \theta_i) - c(\overline{q}, \theta_i) \leq \epsilon$, and the third inequality uses $b_{\text{FPA}}(\theta) \geq \underline{b}'$. Together with (16), this implies that when $b_{\text{FPA}}(\theta) \geq \underline{b}'$, and when $\eta > 0$ and $\epsilon > 0$ are small enough, the bidding and transfer profile satisfies (IC-p).

We now show that the proposed bidding and transfer profile also satisfies (IC-q) whenever $\epsilon > 0$ is small enough. Note that there exists $\alpha_3 < \frac{1}{n}$ such that, for all i and all $q_i' \neq q_i$,

$D_i(b, q_i', q_{-i}) \le \alpha_3$. Indeed, i's probability of winning the auction when bidding $b_i = b_{\text{FPA}}(\theta)$ and $q_i' \neq \overline{q}$ when all other bidders bid $b_{\text{FPA}}(\theta)$ and $\overline{q}$ is strictly lower than $1/n$. Hence,

$$\begin{aligned}\forall i, \forall q_i' \neq \underline{q}, \quad u_i(b, q_i', q_{-i}, \theta_i) - u_i(b, q, \theta_i) &\le \alpha_3(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i)) - \frac{1}{n}(b_{\text{FPA}}(\theta) - c(\overline{q}, \theta_i)) \\ &< \left(\alpha_3 - \frac{1}{n}\right)(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i)) + \frac{1}{n}\epsilon,\end{aligned}$$

where the first inequality uses $D_i(b, q_i', q_{-i}) \le \alpha_3$ and the second uses $c(\overline{q}, \theta_i) - c(\underline{q}, \theta_i) < \epsilon$. Since $\alpha_3 < \frac{1}{n}$, the proposed bidding and transfer profile satisfies (IC-q) whenever ϵ is small enough.

Finally, suppose $\inf_{q_i' \neq q_i'', \widehat{q}} |\ln \gamma(\widehat{q}|q_i') - \ln \gamma(\widehat{q}|q_i'')| > \frac{1}{\epsilon}$ for $\epsilon > 0$ small. Define $\varepsilon(\epsilon) \equiv \frac{1}{\exp\left(\frac{1}{\epsilon}\right)}$, so that $\lim_{\epsilon \to 0} \varepsilon(\epsilon) = 0$. Let

$$\widehat{Q} \equiv \left\{\widehat{q} \in Q : \forall q_i \neq \underline{q}, \ln\left(\frac{\gamma(\widehat{q}|\underline{q})}{\gamma(\widehat{q}|q_i)}\right) > \frac{1}{\epsilon}\right\}$$

to be the set of signals that are more likely under $\underline{q}$ than under any $q_i \neq \underline{q}$.[25] Note that, for all $q_i \neq \underline{q}$, $\text{prob}(\widehat{q}_i \in \widehat{Q}|q_i) < |\widehat{Q}|\varepsilon(\epsilon)$, and that $\text{prob}(\widehat{q}_i \in \widehat{Q}|\underline{q}) \ge 1 - |\widehat{Q}|\varepsilon(\epsilon)$.[26]

Fix a type profile θ with $\max_{i,j} |c(\underline{q}, \theta_i) - c(\underline{q}, \theta_j)| < \eta$, and consider the following bidding and transfer profile $(b, q) = (b_i, q_i)_{i \in I}$ under SA. All bidders submit bid $b_i = b_{\text{FPA}}(\theta)$ and $q_i = \underline{q}$. If $\widehat{q} = (\widehat{q}_i)_{i \in I}$ is such that $I(\widehat{q}) = \{i \in I : \widehat{q}_i \notin \widehat{Q}\}$ has only one element, bidder i with $\widehat{q}_i \notin \widehat{Q}$ pays $\frac{1}{1+\lambda}\frac{\delta}{n}(\overline{V} - \underline{V})$, which is divided even among all other bidders. Otherwise, if $I(\widehat{q})$ is either empty or has more than one element, there are no transfers. Note that this transfer profile is feasible; i.e. $T_i(\widehat{q}) \ge \underline{T}$ for all i and $\widehat{q}$, and $\sum_i T_i(\widehat{q}) = 0$ for all $\widehat{q}$. We now how that, for $\eta > 0$ and $\epsilon > 0$ small enough, this bidding and transfer profile satisfy (IC-p) and (IC-q) as long as $b_{\text{FPA}}(\theta)$ is close enough to r.

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[25]Note that $\widehat{Q}$ is non empty. Indeed, since for every $q_i \neq \underline{q}$, $\gamma(\cdot|q_i)$ F.O.S.D. $\gamma(\cdot|\underline{q})$, and since $\inf_{q_i' \neq q_i'', \widehat{q}} |\ln \gamma(\widehat{q}|q_i') - \ln \gamma(\widehat{q}|q_i'')| > \frac{1}{\epsilon}$, it must be that $\forall q_i \neq \underline{q}$, $\ln\left(\frac{\gamma(\underline{q}|\underline{q})}{\gamma(\underline{q}|q_i)}\right) > \frac{1}{\epsilon}$; i.e., $\underline{q} \in \widehat{Q}$.

[26]Indeed, for all $\widehat{q} \in \widehat{Q}$, we have that $\gamma(\widehat{q}|q_i) < \varepsilon(\epsilon)\gamma(\widehat{q}|\underline{q}) \le \varepsilon(\epsilon)$, and so for all $q_i \neq \underline{q}$, $\text{prob}(\widehat{q}_i \in \widehat{Q}|q_i) < |\widehat{Q}|\varepsilon(\epsilon)$. Similarly, for all $\widehat{q} \notin \widehat{Q}$, there exists $q_i \neq \underline{q}$ such that $\ln\left(\frac{\gamma(\widehat{q}|q_i)}{\gamma(\widehat{q}|\underline{q})}\right) > \frac{1}{\epsilon} \iff \gamma(\widehat{q}|\underline{q}) < \varepsilon(\epsilon)\gamma(\widehat{q}|q_i) \le \varepsilon(\epsilon)$. Hence, $\text{prob}(\widehat{q}_i \in \widehat{Q}|\underline{q}) \ge 1 - |\widehat{Q}|\varepsilon(\epsilon)$.

By the arguments in the proof of Proposition 4, there exists $\alpha_1 < 1$ such that

$$\forall i, \forall b_i' < b_{\mathrm{FPA}}(\theta), \forall q_i', \quad u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) - u_i(b, q, \theta_i) \leq \left(\alpha_1 - \frac{1}{n}\right)(b_{\mathrm{FPA}}(\theta) - c(\underline{q}, \theta_i)) \\ \leq \left(\alpha_1 - \frac{1}{n}\right)(b_{\mathrm{FPA}}(\theta) - \underline{c}(\theta)). \tag{18}$$

Using (10) and $\overline{c}(\theta) - \underline{c}(\theta) < \eta$, we have that

$$\left(\alpha_1 - \frac{1}{n}\right)(b_{\mathrm{FPA}}(\theta) - \underline{c}(\theta)) \leq \left(\alpha_1 - \frac{1}{n}\right)\left(\overline{c}(\theta) - \underline{c}(\theta) + \frac{\delta}{n-1}(\overline{V} - \underline{V})\right) \\ \leq \left(\alpha_1 - \frac{1}{n}\right)\left(\eta + \frac{\delta}{n-1}(\overline{V} - \underline{V})\right).$$

Combining this with (18) we have that

$$\forall i, \forall b_i' < b_{\mathrm{FPA}}(\theta), \forall q_i', \quad u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) - u_i(b, q, \theta_i) \leq \left(\alpha_1 - \frac{1}{n}\right)\left(\eta + \frac{\delta}{n-1}(\overline{V} - \underline{V})\right).$$

Note next that since the bidding and transfer profile are symmetric, we have that $\sum_{\widehat{q}} \mu(\widehat{q}|q) T_i(\widehat{q}|q) = 0$. Hence,

$$\sum_{\widehat{q}} \mu(\widehat{q}|q) T_i(\widehat{q}|q)(1 + \lambda \mathbf{1}_{T_i(\widehat{q})<0}) = -\lambda \mathrm{prob}(\widehat{q}_i \notin \widehat{Q}, \widehat{q}_{-i} \in \widehat{Q}^{n-1}|q) \frac{1}{1+\lambda} \frac{\delta}{n}(\overline{V} - \underline{V}) \\ \geq -\frac{\lambda}{1+\lambda} \mathrm{prob}(\widehat{q}_i \notin \widehat{Q}|q) \frac{\delta}{n}(\overline{V} - \underline{V}) \geq -\frac{\lambda}{1+\lambda} |\widehat{Q}| \varepsilon(\epsilon) \frac{\delta}{n}(\overline{V} - \underline{V}),$$

Since $\alpha_1 < 1$ and since $\varepsilon(\epsilon) \to 0$ as $\epsilon \to 0$, for all $\eta > 0$ and $\epsilon > 0$ small enough we have that

$$\forall i, \forall b_i' < b_{\mathrm{FPA}}(\theta), \forall q_i', \quad u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) - u_i(b, q, \theta_i) \leq \left(\alpha_1 - \frac{1}{n}\right)\left(\eta + \frac{\delta}{n-1}(\overline{V} - \underline{V})\right) \\ \leq \frac{\delta}{n}(\overline{V} - \underline{V}) \\ + \sum_{\widehat{q}} \mu(\widehat{q}|q) T_i(\widehat{q}|q)(1 + \lambda \mathbf{1}_{T_i(\widehat{q}} < 0)).$$

Hence, if $\eta > 0$ and $\epsilon > 0$ are small enough and if $b_{\mathrm{FPA}}(\theta) = r$, the bidding and transfer profile satisfy (IC-p).

Suppose next that $b_{\mathrm{FPA}}(\theta) < r$. As in the proof of Proposition 4, there exists $\alpha_2 < 1$

such that

$$\forall i, \forall b_i' > b_{\text{FPA}}(\theta), \forall q_i', \quad u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) - u_i(b, q, \theta_i) \leq \alpha_2(r - c(\underline{q}, \theta_i)) - \frac{1}{n}(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i)).$$

Assume $\eta < \eta_2$ and let $\underline{b}' = \underline{b} + \frac{\lambda}{1+\lambda}|\widehat{Q}|\varepsilon(\epsilon)\frac{\delta}{n}(\overline{V} - \underline{V})$, with $\eta_2 > 0$ and $\underline{b} < r$ as defined in the proof of Proposition 4. Since $\underline{b} < r$, as since $\lim_{\epsilon \to 0} \varepsilon(\epsilon) = 0$, $\underline{b}' < r$ for all ϵ small enough. Note that, for $b_{\text{FPA}}(\theta) \geq \underline{b}'$, we have that

$$\begin{aligned}
\alpha_2(r - c(\underline{q}, \theta_i)) - \frac{1}{n}(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i)) &\leq \frac{n-1}{n}(b_{\text{FPA}}(\theta) - \overline{c}(\theta)) - \frac{\lambda}{1+\lambda}|\widehat{Q}|\varepsilon(\epsilon)\frac{\delta}{n}(\overline{V} - \underline{V}) \\
\iff b_{\text{FPA}}(\theta) &\geq \alpha_2 r + (1 - \alpha_2)c(\underline{q}, \theta_i) + \frac{n-1}{n}(\overline{c}(\theta) - c(\underline{q}, \theta_i)) \\
&\quad + \frac{\lambda}{1+\lambda}|\widehat{Q}|\varepsilon(\epsilon)\frac{\delta}{n}(\overline{V} - \underline{V}),
\end{aligned}$$

which is always satisfied since $b_{\text{FPA}}(\theta) \geq \underline{b}'$, $\overline{c}(\theta) - c(\underline{q}, \theta_i) \leq \eta$ and since, by (13), $c(\underline{q}, \theta_i) \leq r - \frac{\delta}{n-1}(\overline{V} - \underline{V}) + 2\eta$. Since $\frac{n-1}{n}(b_{\text{FPA}}(\theta) - \overline{c}(\theta)) \leq \frac{\delta}{n}(\overline{V} - \underline{V})$ (by (10)), and since

$$\sum_{\widehat{q}} \mu(\widehat{q}|q)T_i(\widehat{q}|q)(1 + \lambda \mathbf{1}_{T_i(\widehat{q}} < 0)) \geq -\frac{\lambda}{1+\lambda}|\widehat{Q}|\varepsilon(\epsilon)\frac{\delta}{n}(\overline{V} - \underline{V}), \tag{19}$$

it follows that, for $b_{\text{FPA}}(\theta) \geq \underline{b}'$,

$$\begin{aligned}
\forall i, \forall b_i' > b_{\text{FPA}}(\theta), \forall q_i', \quad u_i(b_i', b_{-i}, q_i', q_{-i}, \theta_i) - u_i(b, q, \theta_i) &\leq \frac{\delta}{n}(\overline{V} - \underline{V}) \\
&\quad + \sum_{\widehat{q}} \mu(\widehat{q}|q)T_i(\widehat{q}|q)(1 + \lambda \mathbf{1}_{T_i(\widehat{q}} < 0)).
\end{aligned}$$

Hence, for $\eta > 0$ and $\epsilon > 0$ small enough, the bidding and transfer profile satisfies (IC-p) whenever $b_{\text{FPA}}(\theta) \in [\underline{b}', r]$.

Lastly, we show that the bidding and transfer profile also satisfies (IC-q) whenever ϵ is small enough. Note that, for all $q_i' \neq \underline{q}$

$$\begin{aligned}
&\sum_{\widehat{q}} \mu(\widehat{q}|q_i', q_{-i})T_i(\widehat{q}|q)(1 + \lambda \mathbf{1}_{T_i(\widehat{q}} < 0)) \\
&\leq -\text{prob}(\widehat{q}_i \notin \widehat{Q}, \widehat{q}_{-i} \in \widehat{Q}^{n-1}|q_i', q_{-i})(1 + \lambda)\frac{1}{1+\lambda}\frac{\delta}{n}(\overline{V} - \underline{V}) \\
&\quad + (1 - \text{prob}(\widehat{q}_i \notin \widehat{Q}, \widehat{q}_{-i} \in \widehat{Q}^{n-1}|q_i', q_{-i}))\frac{\lambda}{1+\lambda}\frac{\delta}{n(n-1)}(\overline{V} - \underline{V}) \to -\frac{\delta}{n}(\overline{V} - \underline{V}) \text{ as } \epsilon \to 0.
\end{aligned}$$

Indeed, since for all $q_i' \neq \underline{q}$, $\text{prob}(\widehat{q}_i \notin \widehat{Q}|q_i = q_i') > 1 - |\widehat{Q}|\varepsilon(\epsilon)$, and since $\text{prob}(\widehat{q}_i \in \widehat{Q}|q_i = \underline{q}) \geq 1 - |\widehat{Q}|\varepsilon(\epsilon)$, it follows that

$$\text{prob}(\widehat{q}_i \notin \widehat{Q}, \widehat{q}_{-i} \in \widehat{Q}^{n-1}|q_i', q_{-i})) > (1 - |\widehat{Q}|\varepsilon(\epsilon))^n \to 1 \text{ as } \epsilon \to 0.$$

Combining this with (19), we get that

$$\forall q_i \neq \underline{q}, \quad \sum_{\widehat{q}}(\mu(\widehat{q}|q_i', q_{-i}) - \mu(\widehat{q}|q))T_i(\widehat{q}|q)(1 + \lambda \mathbf{1}_{T_i(\widehat{q}} < 0)) \to -\frac{\delta}{n}(\overline{V} - \underline{V}) \text{ as } \epsilon \to 0.$$

Note further that, by the full support assumption, there exists $\alpha_2 < 1$ such that for all $q_i' \neq \underline{q}$, $u_i(b, q_i', q_{-i}, \theta_i) \leq \alpha_2(b_{\text{FPA}}(\theta) - c(q_i', \theta_i))$. Hence, for all $\eta > 0$ small enough, we have that

$$\begin{aligned}
\forall i, \forall q_i' \neq \underline{q}, \quad u_i(b, q_i', q_{-i}, , \theta_i) - u_i(b, q, \theta_i) &\leq \left(\alpha_2 - \frac{1}{n}\right)(b_{\text{FPA}}(\theta) - c(\underline{q}, \theta_i)) - \alpha_2(c(q_i', \theta_i) - c(\underline{q}, \theta_i)) \\
&\leq \left(\alpha_2 - \frac{1}{n}\right)(b_{\text{FPA}}(\theta) - \overline{c}(\theta)) - \alpha_2(c(q_i', \theta_i) - c(\underline{q}, \theta_i)) + \eta\frac{n-1}{n} \\
&\leq \left(\frac{n\alpha_2 - 1}{n}\right)\frac{\delta}{n-1}(\overline{V} - \underline{V}) - \alpha_2(c(q_i', \theta_i) - c(\underline{q}, \theta_i)) + \eta\frac{n-1}{n} \\
&< \frac{\delta}{n}(\overline{V} - \underline{V}),
\end{aligned}$$

where the first inequality follows from the bound on $u_i(b, q_i', q_{-i}, \theta_i)$ derived above, the second inequality uses $\overline{c}(\theta) - c(\underline{q}, \theta_i) \leq \eta$, the third inequality uses (10), and the last inequality uses $\alpha_2 < 1$ and $\eta > 0$ small enough. Hence, for $\eta > 0$ and $\epsilon > 0$ small enough,

$$\forall i, \forall q_i' \neq \underline{q}, \quad u_i(b, q_i', q_{-i}, , \theta_i) - u_i(b, q, \theta_i) \leq \sum_{\widehat{q}}(\mu(\widehat{q}|q) - \mu(\widehat{q}|q_i', q_{-i}))T_i(\widehat{q}|q)(1 + \lambda \mathbf{1}_{T_i(\widehat{q}} < 0)).$$

Thus, the proposed bidding and transfer profile also satisfies (IC-q). This implies that the bidding and transfer profile with winning bid $b_{\text{FPA}}(\theta)$ is enforceable under SA. Hence, if $\max_{i \neq j} |c(\underline{q}, \theta_i) - c(\underline{q}, \theta_j)| < \eta$, enforceability of $b \geq \underline{b}$ under FPA implies enforceability of b under SA. ■

C Other Evidence of Collusion

This appendix provides additional evidence supporting the inference that procurement auctions let by the MLIT were collusive. It also shows how scoring affected bidder behavior: it made it more difficult to implement designated winner strategies in which bidders where isolated, or in which contracts were disproportionately allocated to incumbents.

Isolated winners. Chassang et al. (2022) and Kawai and Nakabayashi (2022) study the set of auctions held by the MLIT prior to the introduction of scoring auctions. One key finding of that previous work is that winning bids were isolated, and that this is a marker of non-competitive bidding. This pattern is likely an outcome of contract allocation schemes in which one bidder is a designated winner, and others are designated losers.

For each bidder i participating in auction a, define $\Delta^p_{i,a} \equiv \frac{b_{i,a} - \min_{j \neq i} b_{j,a}}{r_a}$, where $b_{i,a}$ is i's cash bid, $\min_{j \neq i} b_{j,a}$ is the lowest cash bid among i's opponents, and r_a is the auction's reserve price. $\Delta^p_{i,a}$ measures the difference between bidder i's own bid and the most competitive bid among i's rivals. When $\Delta^p_{i,a} < 0$, bidder i won the auction; when $\Delta^p_{i,a} > 0$, bidder i lost the auction. Figure C.1 plots the distribution of Δ^p for first-price auctions held by the MLIT in the nine regions of Japan. Across all regions, there is a noticeable missing mass of Δ^p around 0. As we argue in Chassang et al. (2022), these bidding patterns are inconsistent with competitive bidding: when winning bids are isolated, it is a profitable deviation for bidders to increase their bids.

Scoring undermines collusion. We now plot $\Delta^p_{i,a}$ as well as the score difference $\Delta^s_{i,a}$ for scoring auctions. We define $\Delta^s_{i,a}$ as $\Delta^s_{i,a} \equiv \frac{s_{i,a} - \max_{j \neq i} s_{j,a}}{s_a}$, where $s_{i,a}$ is the score of firm i ($= q_{i,a}/s_{i,a}$) and s_a is the lowest acceptable score, $s_a \equiv 100/r_a$. (need to redefine $s_{i,a}$ in code.) $\Delta^s_{i,a}$ is a measure of the % difference in the score between bidder i and its most competitive rival.

Figures C.2 and C.3 plot $\Delta^p_{i,a}$ and $\Delta^s_{i,a}$ for scoring auctions. In Figure C.2, we show that the distribution of $\Delta^p_{i,a}$ no longer exhibits the distinctive missing mass at 0 for many regions. We also find a significant increase in the variance of the distribution. Figure C.3 plots the distribution of $\Delta^s_{i,a}$. The figure shows that the distribution does not exhibit a missing mass at zero for many regions. We see, however, that the density of Δ^s does decrease around $\Delta^s = 0$ for region 1, and to a lesser degree, regions 2 and 7. These changes in the distribution

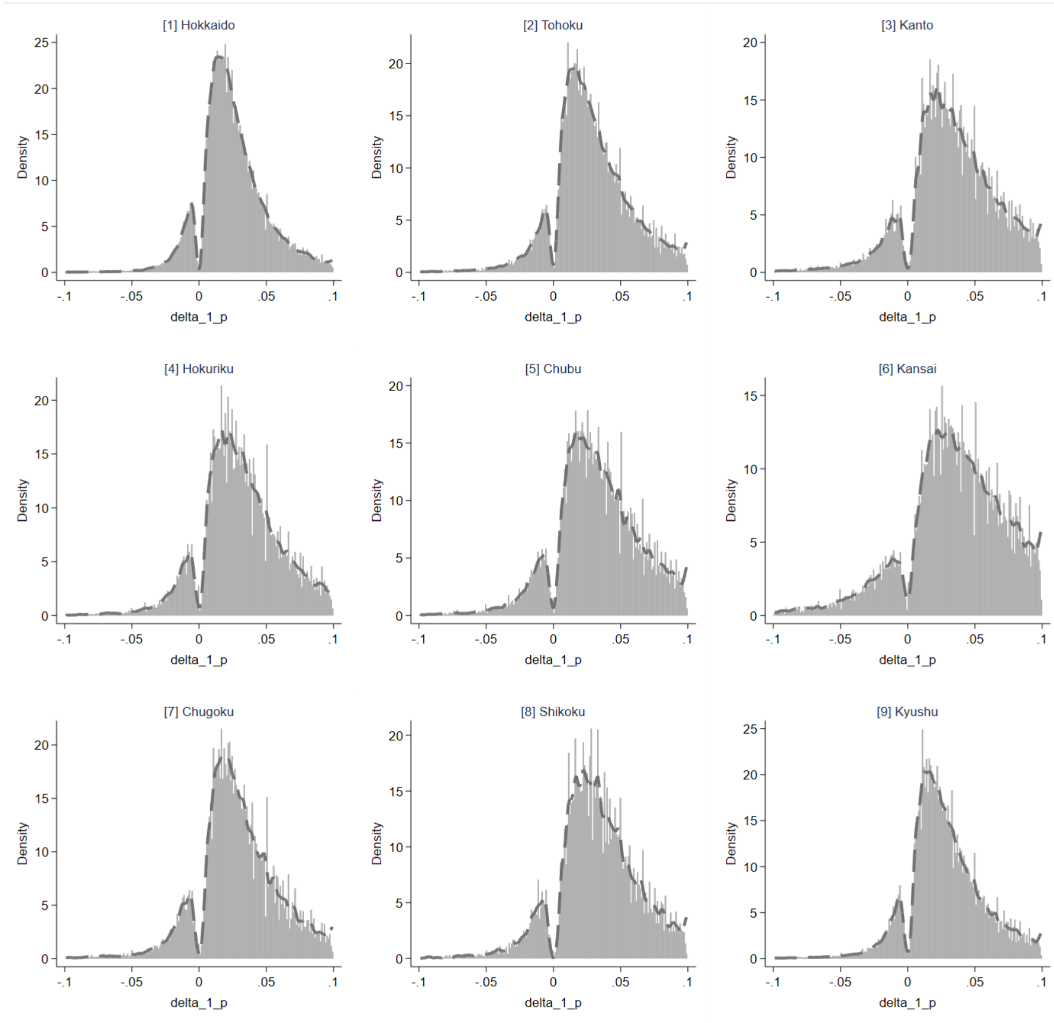

Figure C.1: Distribution of bid differences Δ^p over (bidder, auction) pairs among first price auctions.

suggest that the introduction of scoring auctions forced bidders to change the way in which they collude.

Scoring undermines allocation to incumbents. In Kawai et al. (2023b), we develop a test of collusion based on whether or not marginal winners are more likely to be incumbents than marginal losers. In particular, for each bidder i in auction a, define $I_{i,a}$ to be 1 if bidder i was the winner of the previous auction. We define $I_{i,a}$ to be 0 if bidder i was not the winner or did not participate in the previous auction. In Kawai et al. (2023b), we considered the conditional expectation of $I_{i,a}$ as a function of Δ^p, $\mathbb{E}[I_{i,a}|\Delta^p]$. We showed that $\mathbb{E}[I_{i,a}|\Delta^p = 0^+]$ and $\mathbb{E}[I_{i,a}|\Delta^p = 0^-]$ must be equal to each other under competition, and that deviation from

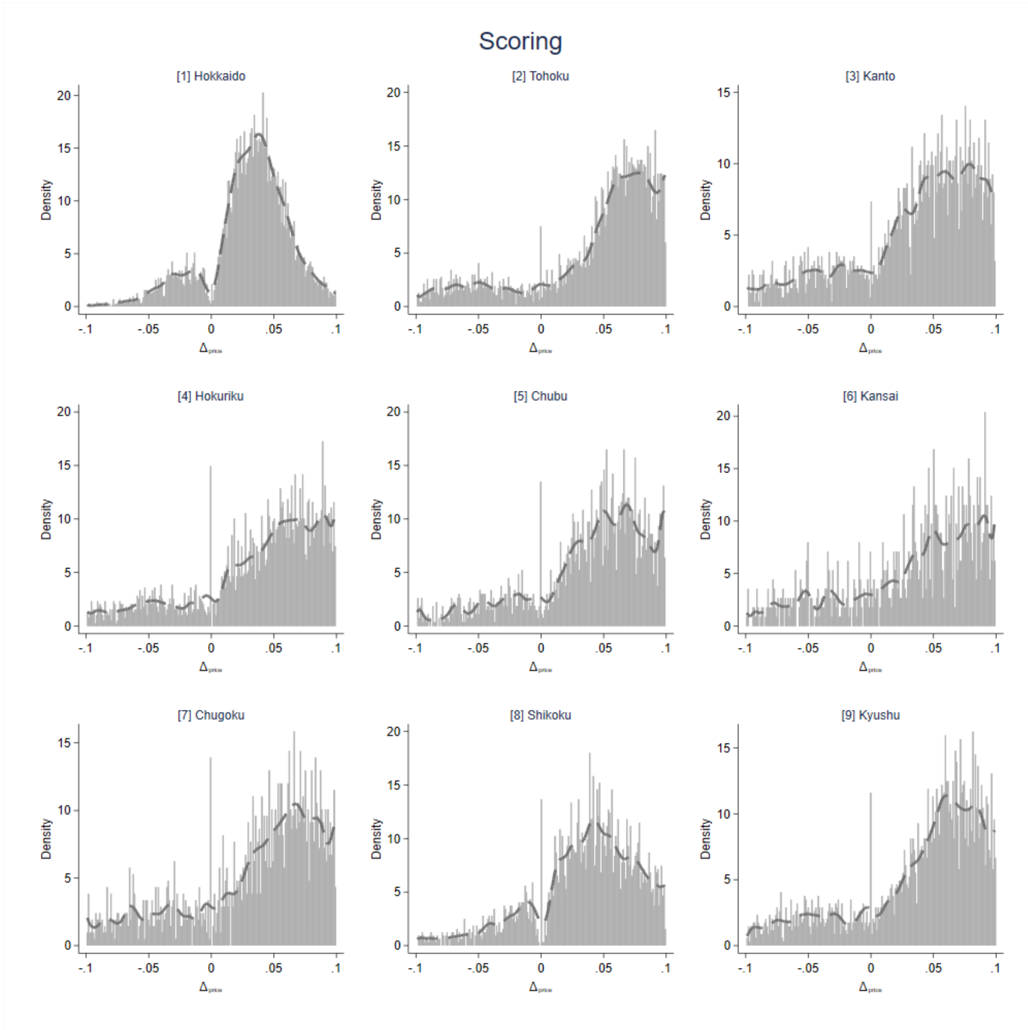

Figure C.2: Distribution of bid differences Δ^p over (bidder, auction) pairs among scoring auctions.

equality is evidence of collusion.

Figure C.4 plots the distribution of price differences Δ^p and score differences Δ^s (top panels) as well as the probability of being an incumbent as a function of these differences (bottom panels). The top left panel plots the histogram of Δ^p for first price sealed bid auctions, the top middle panel plots the histogram of Δ^p for scoring auctions, and the top right panel plots the histogram of Δ^s for scoring auctions.

The bottom panels of Figure C.4 plot the expectation of $I_{i,a}$ as a function of Δ^p or Δ^s. The left panel corresponds to first-price auctions, the middle and right panels correspond to scoring auctions. Each dot in the figure corresponds to a bin average of the underlying

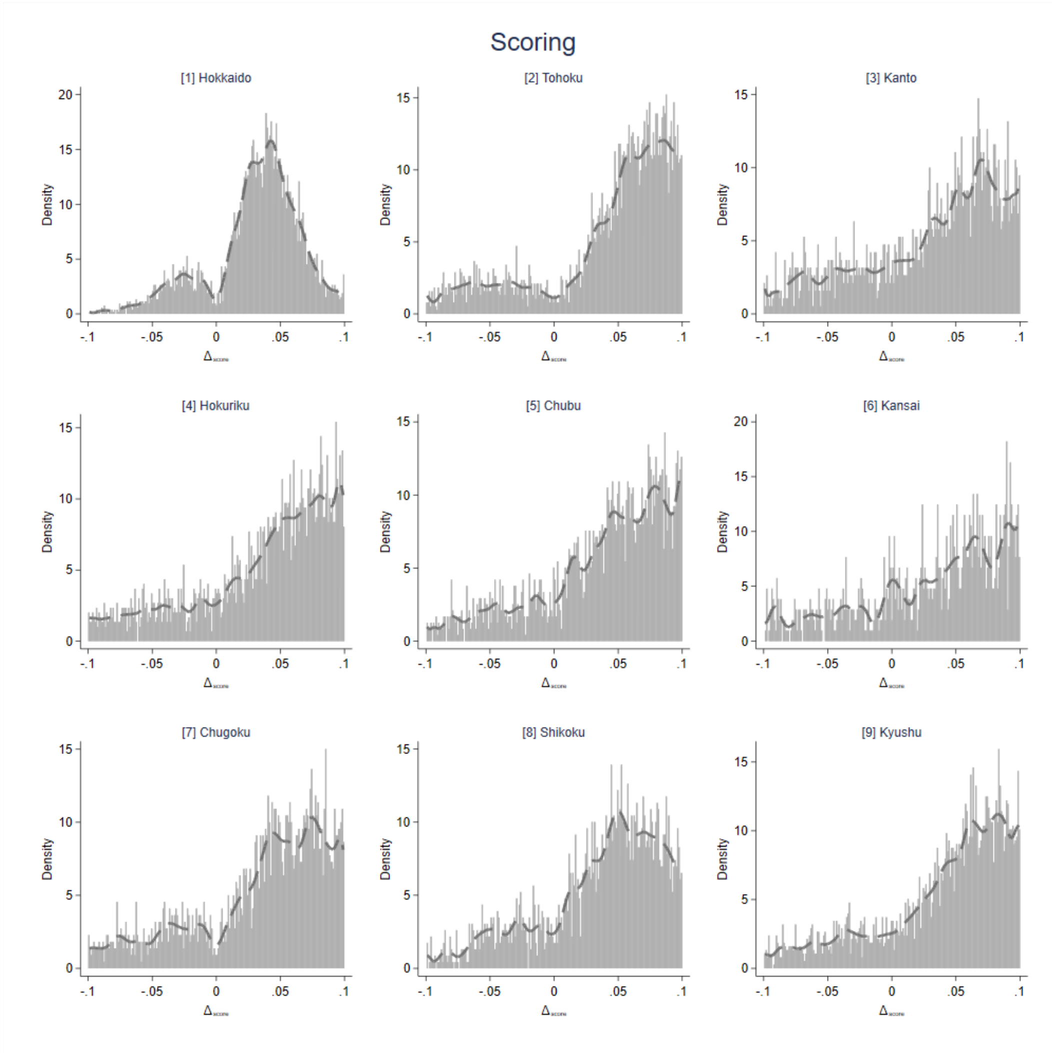

Figure C.3: Distribution of bid differences Δ^s over (bidder, auction) pairs among scoring auctions.

outcome variable, which is either 0 or 1. We can see from the bottom left panel of the figure that the marginal winners, i.e., bidders with $\Delta^p_{i,a} \in [-\epsilon, 0)$ for a small positive ϵ, are much more likely to be incumbents than the marginal losers (those with $\Delta^p_{i,a} \in (0, \epsilon]$). This suggests that the bidders in this sample are colluding by allocating projects to the incumbents. Interestingly, the bottom center panel of the figure suggests that bidders continued to collude in this manner under scoring auctions as well. However, the bottom right panel suggests that the winning probability is close to a half conditional on Δ_s being within a few percentage of zero. This suggests that there is randomness in the quality evaluation such

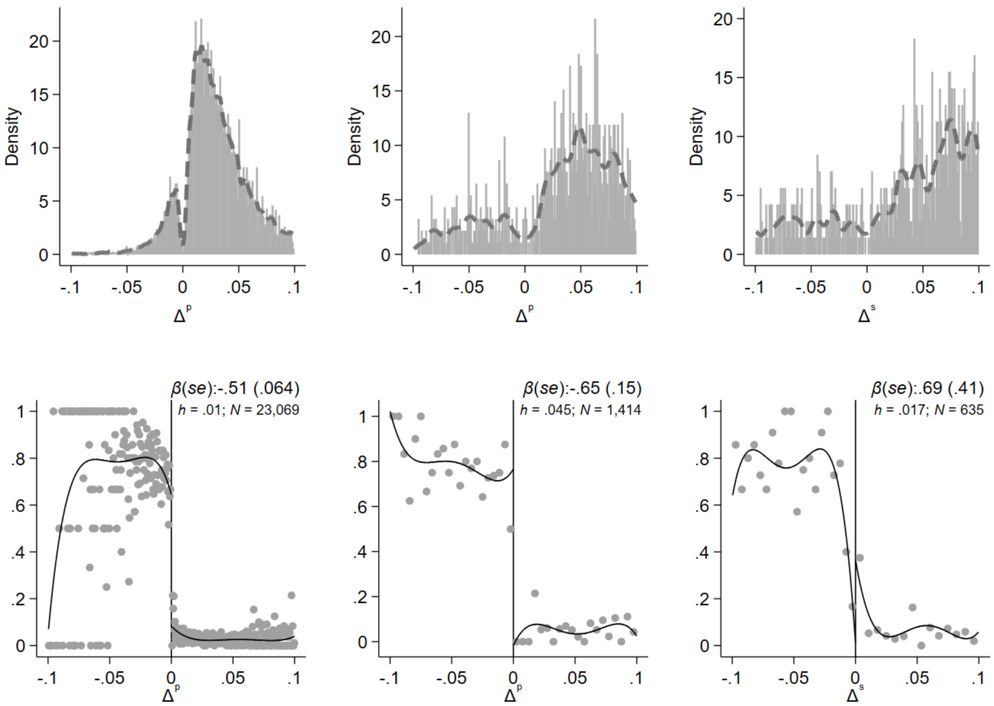

Figure C.4: Probability of being incumbent and distribution of bid differences.

that bidders cannot fully control who wins and who loses.

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	(1)	(2)
Constant		0.9658
		(0.0023)
1st Decile	0.0415	0.0705
	(0.0232)	(0.0186)
2nd Decile	0.0145	0.0052
	(0.0115)	(0.0077)
3rd Decile	-0.0237	-0.0286
	(0.0078)	(0.0054)
4th Decile	-0.0303	-0.0401
	(0.0057)	(0.0044)
5th Decile	-0.0391	-0.0406
	(0.0038)	(0.0031)
6th Decile	-0.0291	-0.0293
	(0.0032)	(0.0028)
7th Decile	-0.0311	-0.0286
	(0.0034)	(0.0029)
8th Decile	-0.0274	-0.0222
	(0.0026)	(0.0024)
9th Decile	-0.0167	-0.0130
	(0.0033)	(0.0028)
10th Decile	-0.0200	-0.0208
	(0.0169)	(0.0114)
N		86,858

Outcome variable is the winning bid. Deciles are defined based on the winning bid of auctions in which a firm participates pre-2004.
*, **, and *** respectively denote significance at the 10%, 5%, and 1% levels.

Table 6: Regression of Winning Bid: Scoring Auctions vs. First Price Auctions