Detecting Large-Scale Collusion in Procurement Auctions*

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Abstract

We document evidence of widespread collusion among construction firms in Japan using a novel dataset covering most of the construction projects procured by the Japanese national government. Our dataset contains information on about 42,000 auctions whose award amount sums to about $40 billion. We identify collusion by focusing on rebids that occur for auctions in which all (initial) bids fail to meet the secret reserve price. We identify more than 1,000 firms whose conduct is inconsistent with competitive behavior. These bidders were awarded about 15,000 projects, or about 37% of the total number of projects in our sample.

Key words: Collusion, Procurement Auctions, Antitrust
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1 Introduction

One of the central themes of competition policy is to deter, detect, and punish collusion. While there is almost universal agreement among economists that collusion among firms

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is socially undesirable, firms often have private incentives to engage in collusive behavior absent regulatory sanctions. Therefore, it is crucial to ensure that the antitrust agencies have the authority and the resources to detect and punish collusion in order to promote competition among firms. To the extent that collusive activities remain undetected or unpunished, collusion may become the norm rather than the exception, with potentially large detrimental effects on the economy.

In this paper, we document evidence of widespread collusion among procurement auctions in Japan. A key feature of the auctions that we study is that there is rebidding after an unsuccessful auction in which no bid meets the secret reserve price. Using this feature, we construct several tests of competition and apply them to data. Our dataset covers most of the construction projects procured by Japan's national government between April 2003 and December 2006, consisting of about 42,000 auctions and award amount totaling about $40 billion. We find evidence of collusion that persists across regions, across types of construction projects and across time. In particular, we find more than $1,000$ construction firms for which we reject the null hypothesis of competitive behavior. These bidders were awarded a total of 15,583 auctions, or 37.1% of the total number of auctions in our sample. The total award amount of these auctions is about $18.6 billion, or 44.2% of the total award amount in our sample.

Our tests for detecting collusion exploit bidding patterns that result from preallocating auctions to designated bidders. If there is a bidding ring and the project is preallocated to one of its members, the designated winner can be expected to submit the lowest bid in the initial auction as well as in the reauctions, if they occur. The designated losers, on the other hand, will avoid undercutting the bid of the designated winner in the initial auction as well as in the reauctions. Preallocation of auctions thus leads to persistence in the identity of the lowest bidder across the initial auction and the reauctions. The tests of competition that we propose in this paper leverage this idea.

In order to construct formal tests with correct size (i.e., controls for false positives), the tests need to distinguish between persistence in the rank order of bidders that results from collusive preallocation and persistence that results from simple cost heterogeneity among competitive bidders. The key step for constructing formal tests is to properly account for the range of possible bidding patterns that can arise under competitive bidding with asymmetric bidders. We consider two models of competitive behavior. The first one is expected profit maximization with rational expectations in a one-shot (i.e., not repeated) game.[1] A necessary condition of expected profit maximization is that losers of the initial auction should not profitably gain from bidding more aggressively in the reauctions. Although we do not observe bidder costs, we can bound the counterfactual profits from bidding more aggressively using bounds on costs derived by the restriction that bids in subsequent reauctions are above the bidder's costs. Given that the time between the initial auction and all subsequent auctions is very short in our setting, using subsequent bids to bound costs seems reasonable. By combining revealed preference restrictions with upper bounds on costs, we construct a test of competition, rejection of which implies that bidding is inconsistent with profit maximization in the one-shot game. Specifically, rejection of the test suggests complementary bidding by designated losers.

While violation of profit maximization in the one-shot game is suggestive of collusive non-Markov equilibria, we also consider a second model that captures a different aspect of competitive bidding. Our model is motivated by the fact that, while large and systematic errors are uncommon, small errors in the actions of players that deviate from optimal play are prevalent (See e.g, McKelvey and Palfrey, 1995 and Samuelson, 2005). These errors are particularly salient in the bidding context since formulation of the final bid is often a complex and lengthy process with room for many random factors to affect the final bid. Absence of any such idiosyncratic errors suggests coordinated bidding.

In order to incorporate small errors to the bidding strategy, we consider a second benchmark of competitive behavior that extends the standard model of bidding. In particular, we let small idiosyncratic shocks to affect bidders' bids and posit that, in the absence of coordinated bidding, there exists one element of these shocks that are independent of the rivals' bids. Note that our assumption does not necessarily imply that bidders are making mistakes that are very costly or that bidders' strategies are far from equilibrium play. The shocks can be small optimization errors around the optimum.

We now briefly summarize the empirical patterns that contradict our benchmarks of competitive behavior. The first pattern is how the losers of the initial auction bid in the reauction. Letting $i(1)$ and $i(2)$ denote the lowest and the second lowest bidders of the initial auction, respectively, we find that $i(2)$ loses to $i(1)$ in the reauction more than 95% of the time. More specifically, we find that there are hundreds of auctions in which $i(2)$ is outbid by $i(1)$ in the reauction by a tiny margin, but almost none in which $i(1)$ is outbid by $i(2)$ in the reauction. This pattern implies that $i(2)$ can win substantially more by bidding

[1] Here, the one-shot game refers to the game that consists of the initial auction and the subsequent reauctions for the same project. The one-shot game does not include lettings for other projects.

only slightly more aggressively in the reauction, which in turn, suggests that $i(2)$ is not playing a best response to $i(1)$ in the reauction. Motivated by this pattern, we formally test whether or not losers of the initial auction can increase profits by bidding more aggressively in the reauction. Using bids in subsequent reauctions to bound costs, we find, for example, that bidders who are outbid by $i(1)$ in the initial auction by less than 1% of the reserve price would be able to increase profits by more than 1.7 million yen by uniformly reducing their bid in the reauction by 2%. These bidding patterns are inconsistent with profit maximization in the one-shot game and suggests complementary bidding.

A second related bidding pattern that we document is what appears to be a kink in the distribution of Δ_{12}^2, the bid difference between $i(1)$ and $i(2)$ in the reauction. When we plot the distribution of Δ_{12}^2, we find that the shape of the distribution has a distinctive kink at zero. The kink in the distribution implies that the probability that $i(2)$ outbids $i(1)$ remains bounded away from 0.5 even as $|\Delta_{12}^2|$ approaches zero, which is inconsistent with our second benchmark of competitive behavior. Our second benchmark of competition posits that there exists an idiosyncratic component to the bids of each bidder, an implication of which is that the probability that $i(2)$ outbids $i(1)$ must converge to 0.5 as $|\Delta_{12}^2|$ approaches zero.

The third pattern that we document is the difference between firms that marginally outbid $i(1)$ in the reauction and those that are marginally outbid by $i(1)$ in the reauction. In particular, we find that the average winning bids of past auctions in which the former set of bidders participate are significantly lower than that of auctions in which the latter set of bidders participate. Under the assumption that there exists an idiosyncratic component to the bids, bidders who marginally outbid $i(1)$ in the reauction should, on average, look exactly the same as those who are marginally outbid by $i(1)$. Hence, this finding is also inconsistent with our second benchmark. On the other hand, this pattern is very much consistent with the idea that colluding bidders refrain from outbidding $i(1)$ while competitive bidders are under no such constraints.

Overall, the results of our tests suggest that collusion is widespread in our sample. For example, among the 1,000 largest firms in our dataset, we reject the null of competition for 613 firms. In total, we reject the null of competition for 1,066 firms. The number of auctions won by these firms totals 15,583, or 37.1$\%$ of the total number of auctions in our sample.

The pervasiveness of collusion among construction firms that we document may have significance for the broader economy. While our dataset accounts only for public construction projects procured at the national level, firms that we identify as uncompetitive are also active in prefectural and municipal procurement auctions. Evidence of collusion among national-level auctions suggests that collusion may be wide-spread among all public construction projects. The total value of public construction projects in Japan (which includes projects procured by both local and national governments) is about \$200 billion per year and accounts for about 20% of total government expenditure. It amounts to about 4% of Japan's GDP.

More generally, the scale of collusion that we document in the paper highlights the importance of rigorous enforcement of competition policy. Our findings lend support to the view that, absent competition policy, collusion can be widespread and affect a significant portion of an economy – as opposed to the view that collusion occurs sporadically and only under a limited set of circumstances.[2] Our results seem to indicate that entrusting the antitrust agencies with the authority and resources to detect and punish collusion have important aggregate-level implications.[3]

The approach that we use to detect collusion may be useful in other contexts as well. While the detection methods that we propose in this paper are tailored to the institutional setting of Japan, rebidding is a common feature of procurement auctions.[4] Examples in which the auctioneer holds multiple auctions for the same object include timber auctions by the U.S. Forest Services, procurement auctions by the U.S. State DoT, and U.S. offshore gas and oil lease auctions.[5] Although the exact institutional features of the auctions differ from setting to setting (e.g., the time between the initial auction and the reauction, whether the bids of the initial auction are made public, etc.), the approach in our paper may provide a useful starting point when screening for collusion in similar settings.

[2]Porter (2005), for example, expresses a view that is close to the former: "In any market, firms have an incentive to coordinate their decisions and increase their collective profits by restricting output and raising market prices." For the latter view, see, e.g., Schmalensee (1987) and his description of Demsetz (1973) and Demsetz (1974). In his description of the Differential Efficiency Hypothesis, Schmalensee (1987) writes, "Effective collusion is rare or nonexistent." See also Shleifer (2005), p. 440.

[3]There is a debate on whether or not competition policy is effective at increasing total factor productivity (see, e.g., Buccirossi et al., 2013).

[4]Although auctions with rebidding is not an optimal mechanism for the auctioneer when bidders are competitive, one potential advantage is that it makes for screening for collusion easier.

[5]See McAfee and Vincent (1997) for a description of reauctions in U.S. Forest Service timber sales, Ji and Li (2008) for DoT auctions in Indiana, and Porter (1995) for offshore gas and lease auctions. Ji and Li (2008) examine DoT auctions in which the auctioneer sets a secret reserve price, and there are multiple auctions for the same project when none of the bids meets the secret reserve price. In their sample, they find that about 12.5% of lettings have two rounds of bidding. In his analysis of wildcat tracts, Porter (1995) reports: "A total of 233 high bids, or 10 percent were rejected on these tracts. On the tracts with rejected bids, 47 percent were subsequently reoffered."

Lastly, the findings in this paper shed light on the internal organization of bidding rings that can potentially explain the puzzling bidding patterns documented in our companion paper, Chassang et al. (2021). Chassang et al. (2021) document a missing mass of almost tied bids in the initial auction using the same dataset as ours. In the current paper, we find persistence in the identity of the lowest bidder across the initial auction and subsequent reauctions which suggests that bidding rings preallocate auctions to designated winners and that designated losers place complementary bids. Given that complementary bids are typically communicated from the designated winners to the designated losers, the missing mass of almost tied bids is likely to be related to the way in which complementary bids are communicated between the bidders. If, for fear of leaving a paper trail, communication between the cartels are carried out verbally, it seems reasonable to expect that the instructions regarding the complementary bids are coarse, i.e. unlikely to be specified up to the penny. If the designated bidder specifies a round number above which designated losers should bid while the designated winner places a bid just below it, it results in a very small, but distinct missing mass of almost tied bids as documented in Chassang et al. (2021).

1.1 Related Literature

This paper contributes to the empirical literature on detecting collusion in auctions.[6] Existing empirical studies of collusion tend to take advantage of known episodes of cartel activity: e.g., paving in highway construction in Nassau and Suffolk counties (Porter and Zona, 1993); school milk delivery in Ohio (Porter and Zona, 1999); school milk delivery in Florida and Texas (Pesendorfer, 2000); collectible stamps in North America (Asker, 2010a); and supply of asphalt in Quebec (Clark et al., 2018). While none of our analysis requires information on known bidding rings, it is still useful to study the bidding behavior of known cartels for validation purposes. Online Appendix IV contains an analysis of the four bidding cartels that were prosecuted by the Japan Fair Trade Commission (JFTC).

Another strand of literature tests for collusion in the absence of any prior knowledge of bidder conduct. Examples include seal coat contracts in three states in the U.S. Midwest (Bajari and Ye, 2003); timber auctions in U.S. and Canada (Baldwin et al., 1997; Athey et al., 2011; Schurter, 2017); offshore gas and oil lease auctions (Hendricks and Porter, 1988; Haile et al., 2012); roadwork contracts in Italy (Conley and Decarolis, 2016); LIBOR (Abrantes-Metz et al., 2012; Snider and Youle, 2012); public-works consulting in

[6]For a brief survey, see Asker et al. (2010b). For a more comprehensive study, see, e.g., Porter (2005), Harrington (2008) and Marshall and Marx (2012).

Okinawa, Japan (Ishii, 2009); municipal public works in Ibaraki (Chassang and Ortner, 2019); and municipal and national public works in Japan (Chassang et al., 2021). Ishii (2009) studies 175 auctions of design consultant contracts in Naha, Okinawa and analyzes how exchange of favors can explain the winner of the auctions. Chassang and Ortner (2019) study theoretically how the introduction of minimum prices affects cartel behavior and document evidence consistent with their theoretical predictions using data from municipalities in Ibaraki.[7] Chassang et al. (2021) follow up on our work by studying bidding behavior using both National and municipal auctions from Japan and find additional evidence of collusion. In particular, Chassang et al. (2021) document a missing mass of almost tied bids.

The paper is also related to the literature on identification and estimation of incomplete models (See, e.g., Tamer (2010) for a survey), and in particular, to the work of Haile and Tamer (2003). In Haile and Tamer (2003), they partially identify the distribution of bidders' values in English auctions using the restriction that bidders do not bid above their values. Similarly, we use the idea that the costs of bidders can be bounded above by their bids in reauctions for conducting a test of collusion. The bounds on costs allow us to put bounds on the profits from playing alternative bidding strategies.

Finally, this paper is related to the literature on sequential auctions. McAfee and Vincent (1997) study the problem of a seller who can post a reserve price but cannot commit never to attempt to resell an object if it fails to sell. Skreta (2015) solves for the seller's optimal mechanism with no commitment and shows that multiple rounds of either first- or second-price auctions with optimally chosen reserve prices maximize the seller's revenue when the bidders are ex-ante identical. The auctions in our dataset have the feature that the seller cannot commit never to resell but can commit to the same secret reserve price. Ji and Li (2008) structurally estimate a model of multi-round procurement auctions with a secret reserve price using data on procurement auctions let by the Indiana DoT. The Indiana DoT also maintains the same secret reserve price throughout the multiple rounds as in our setting. Ji and Li (2008) recover the private cost distributions of the bidders assuming that the observed bids are competitive.

[7] For a more general overview of bidding rings among procurement firms in Japan, see McMillan (1991). See, also, Ohashi (2009), who discusses how the change in auction design in Mie prefecture affected collusion.

2 Institutional Background and Bidding Behavior

Auction Mechanism The auction format used in our data is a first-price sealed-bid (FPSB) auction with rebidding. The auction mechanism is exactly the same as the standard FPSB auction as long as the lowest bid is below the secret reserve price, in which case, the lowest bidder becomes the winner with a price equal to the lowest bid, and the auction ends. If none of the bids is below the reserve price, however, the buyer publicly announces the lowest bid to all the bidders and solicits a second round of bids. The buyer reveals only the lowest bid and none of the other bids (the identity of the lowest bidder and the secret reserve price are not revealed). The second-round bidding typically takes place 30 minutes after the initial round, with the same set of bidders and the same secret reserve price. This means that when bidding in the second round, the bidders know that the secret reserve price is lower than the lowest first-round bid.

The second round proceeds in the same manner as the initial round; if the lowest bid is below the reserve price, the auction ends, and the lowest bidder wins. Otherwise, the buyer reveals the lowest second-round bid to the bidders, and the auction goes to the third round. The third round is the final round. If no bid meets the reserve price in the third round, bilateral negotiation takes place between the buyer and the lowest third-round bidder. The same secret reserve price is used in all three rounds.

Bidding takes place online in all three rounds, and the identity of the bidders is not public at the time of bidding. The reserve price, the identity of the bidders, and all the bids in each round are made public after the auction ends.[8]

Reserve Price The auctioneer computes the reserve price of each auction by breaking down each project into specific procedures, each of which is then converted into a list of required input quantities. For each input, the itemized price is computed by multiplying the total input quantity by its unit price. The reserve price is obtained by summing the itemized prices. Given that both the formula for converting procedures to input quantities and the unit prices are published by the auctioneer, guessing the reserve price fairly accurately is often not very difficult.[9] However, there is rounding of itemized prices in the process of

[8]The fact that all bids from all rounds are made public may facilitate collusion by making it easier for cartels to detect deviations.

[9]There is not much room for the auctioneer to exercise discretion in setting the reserve price. In fact, there are many commercial softwares that estimate the reserve price fairly accurately given information on input quantities.

computing the reserve price, which makes the reserve price random from the perspective of bidders.

Participation As is the case in many countries, participation in procurement auctions in Japan is not fully open. A contractor that wishes to participate must first go through screening to be pre-qualified. Pre-qualification occurs for each (region, project type, project size category) triplet. An example of a triplet is (Hokkaido, Civil Engineering, Projects worth less than 60 million yen). There are a total of 9 regions (e.g., Hokkaido, Tohoku, etc.) and about 20 project types (Civil Engineering, Paving, Landscaping, etc.) in total. Each project type in each region is further divided into 1 to 4 project size categories.[10] While a given firm can be pre-qualified in multiple regions and for multiple project types, a given firm cannot qualify for more than one project size category for a given region and project type. This restriction ensures that rivalry is limited to mostly similarly sized firms. There are about 300 triplets in total. For each triplet, there are about 230 firms that bid at least once on that triplet. This number falls to about 65 if we count only firms that bid at least 10 auctions for a given triplet.

In about 80% of the auctions, the MLIT restricts participation further by inviting a subset of pre-qualified firms (typically about 10 firms) to the auction.[11] Firms that are not invited are not eligible to participate even if the firm is pre-qualified for the project. In only about 20% of the auctions, the MLIT makes a public call for a tender. All pre-qualified bidders are eligible to participate in these auctions.

While the number and the identity of the participating bidders are not disclosed to any of the bidders beforehand, bidders can often predict fairly accurately the set of participants by observing who comes to the on-site briefing.

Collusive Behavior In principle, bidding rings can be organized in a variety of ways depending on whether or not members engage in side payments, whether explicit communication between the members is feasible, etc. Whatever the exact arrangement, however, a very common feature of bidding rings is that the ring picks a predetermined winner in

[10] For example, civil engineering is typically segmented into four project size categories. The largest one corresponds to projects above 720 million yen, the second one corresponds to those between 300 and 720 million, the third corresponds to those between 60 and 300 million, and the last corresponds to those less than 60 million.

[11] The MLIT typically decides which firms to invite to the auction based on considerations such as proximity to the construction site and past performance of firms on similar projects.

advance and that the rest of the ring members help that predetermined winner win. The existing evidence indicates that bidding rings in the construction industry in Japan are often organized in this manner.[12] The detection method that we propose in the paper exploits this feature.

A few other documented features of prosecuted bidding rings in the Japanese construction industry are worth mentioning: First, the designated winner alone typically incurs the cost of estimating the project cost.[13] Estimating the project cost can be quite expensive, and the non-designated bidders typically avoid incurring this cost.[14] Note that this makes it risky for a non-designated bidder to accidentally win the auction. Second, the designated winner of a bidding ring would often communicate with other members in advance how they should bid in each of the three rounds (as opposed to communicating how they should bid in just the first round).[15] Lastly, most prosecuted bidding rings consist of many firms, often including more than 20+ firms.[16] Some of the detected cartels were close to being all-inclusive while others were partial.[17]

3 Data

We use a novel dataset of auctions for public construction projects obtained from the Ministry of Land, Infrastructure and Transportation (MLIT), the largest single procurement buyer in Japan. The dataset spans April 2003 through December 2006 and covers most of the construction work auctioned by the Japanese national government during this period. After dropping scoring auctions, unit-price auctions, and those with missing or mistakenly

[12]See, for example, a report issued by the Japan Federation Bar Association (JFBA), which studies criminal bid-rigging cases (JFBA, 2001).

[13]See, e.g., the criminal bid-rigging case regarding the construction of a sewage system in Hisai city (Tsu District Court, No. 165 (Wa), 1997); the bid-rigging case regarding the construction of a waste incineration plant in Nagoya city (Nagoya District Court, No. 1903 (Wa), 1995); etc. Based on facts that became clear in these cases, the JFBA concludes that the project estimation costs are borne only by the designated winner in many bidding rings. (JFBA 2001, p20)

[14]Estimating the project cost involves understanding the specifications of the project, assessing the quantity and quality of materials required, negotiating prices for construction material and arranging for available subcontractors. These costs are often quite substantial.

[15]See JFBA (2001), p19 and JFTC (2010b), pp.10-11.

[16]It is not uncommon for bidding rings to consist of many members. See Asker (2010a).

[17]Our tests of collusion do not require us to specify whether the cartels are all-inclusive or partial. Our null is that all of the bidders are competitive. Rejection of the null can imply either partial or all-inclusive cartels, although the tests are likely to have more power against all-inclusive cartels. The distinction between all-inclusive cartels and partial cartels is important in the tests proposed in Porter and Zona (1993) and Baldwin et al. (1997).

Concluding Round	(R)eserve Yen M. (1)	(W)inbid Yen M. (2)	(W)/(R) (3)	Lowest bid / Reserve Round 1 (4)	Round 2 (5)	Round 3 (6)	# Bidders (7)	N (8)
1	103.401	97.232	0.927	0.927	-	-	9.85	33,575
	(245.72)	(235.62)	(0.085)	(0.085)			(2.60)	80.0%
2	86.214	83.594	0.965	1.056	0.965	-	9.89	7,138
	(199.24)	(194.51)	(0.033)	(0.075)	(0.033)		(2.42)	17.0%
3	62.601	60.646	0.963	1.143	1.071	0.963	9.41	1,249
	(160.66)	(158.34)	(0.034)	(0.113)	(0.089)	(0.034)	(2.26)	3.0%
All	99.263	93.823	0.935	0.956	0.981	0.963	9.85	41,962
	(236.46)	(227.29)	(0.079)	(0.103)	(0.060)	(0.034)	(2.56)	100.0%

Note: The first row corresponds to the summary statistics of auctions that ended in the first round; the second row corresponds to auctions that ended in the second round; and the third row corresponds to auctions that went to the third round, including 39 auctions that were determined by bilateral negotiation. The last row reports the summary statistics of all auctions. The numbers in parentheses are the standard deviations. First and second columns are in millions of yen.

Table 1: Summary Statistics

recorded entries, we are left with 41,962 auctions with a total award amount of more than \$39 billion.[18]

The data include information on all bids, bidder identity, the secret reserve price, auction date, location of the construction site, and the type of project.[19] The data also contain information on whether the auction proceeded to the second round or the third round, as well as all the bids in each round. Table 1 provides summary statistics of the data. In the table, we report the reserve price of the auction (Column (1)), the winning bid (Column (2)), the ratio of the winning bid to the reserve price (Column (3)), the lowest bid in each round as a percentage of the reserve price (Columns (4)-(6)), and the number of bidders (Column (7)). The sample statistics are reported separately by whether the auction concludes in Round 1, Round 2, or proceeds to Round 3.[20]

The first and second columns of the table show that the average reserve price of the auctions is about 99 million yen, and the average winning bid is about 94 million yen. The

[18]Samples with missing or mistakenly recorded entries each account for 3.0% of the entire dataset. Scoring auctions account for 8.0%.

[19]Construction projects are divided into about 20 types of construction work, such as civil engineering, architecture, bridges, paving, dredging, painting, etc.

[20]The sample that proceeds to Round 3 includes 39 auctions that were decided by bilateral negotiation.

project size of auctions with two or more rounds tend to be smaller than that for auctions that conclude in Round 1.[21] In Column (3), we find that the winning bid ranges between 93% and 97% of the reserve price. In the next three columns, we report the lowest bid in each round as a fraction of the reserve price. Note that for auctions that conclude in the first round, the number in Column (4) is equal to the number in Column (3). For auctions that conclude in the second or third round, the numbers reported in Column (4) are higher than unity by construction. Column (8) reports the sample size. We find that 20.0% of the auctions go to the second round, and 3.0% advance to the third round. Some bidders who bid in Round 1 may decide not to bid in Round 2 or Round 3.[22]

Table 2 reports how the rank of the bidders changes from the first round to the second round for all auctions that proceed to the second round with five or more participants ($N = 8,016$). The (i, j) element of the Table corresponds to the probability that a bidder submits the j-th lowest bid in the second round, conditional on submitting the i-th lowest bid in the first round; i.e., $\Pr(j\text{-th lowest}|i\text{-th lowest})$. Thus, the diagonal elements correspond to the probability that a given bidder remains in the same rank in both rounds. Note that the horizontal sum of the probabilities is one.

We find that in 96.64% of the auctions, the lowest bidder in the first round remains the lowest bidder in the second round. The probability that a trailing bidder from the initial auction outbids the lowest first-round bidder is very low. For example, the conditional probability that a second-lowest bidder in Round 1 becomes the lowest bidder in Round 2 is only 1.62%. Note, also, that the diagonal elements other than the $(1, 1)$ element are much smaller: the probability that the second-lowest bidder in the first round remains the second-lowest bidder is just 26.68%. There is very strong persistence in the identity of the lowest bidder, but not necessarily for other positions.

Table 3 reports the summary statistics of the bidders in our data set. We group the firms into five by the total number of auctions the firm participates in our sample and report the summary statistics by group. The top row corresponds to the summary statistics for the 34 largest firms that participate in more than 500 auctions. The second row corresponds to the set of 658 firms that participate in 100 to 500 auctions, and so on.

[21] Auctions for small projects have a higher bid-to-reserve ratio and tend to have multiple rounds of bidding. These results suggest that bidding is more competitive for larger projects. This result is consistent with the theory of Rotemberg and Saloner (1986). When the project size is larger, the temptation to deviate for non-designated bidders becomes larger. In order to sustain collusion, the lowest bid on very large projects need to be sufficiently lowered. More results on this point are available upon request.

[22] Online Appendix I contains a detailed analysis of attrition.

		Round 2				
		1	2	3	4	5+
	1	96.64%	1.63%	0.62%	0.27%	0.82%
	2	1.62%	26.68%	18.61%	13.32%	39.76%
Round 1	3	0.55%	18.84%	18.54%	13.91%	48.16%
	4	0.38%	14.23%	15.93 %	15.43 %	54.03%
	5+	0.13%	6.75%	9.23%	10.39%	73.50%

Note: The $(i{,}j)$ element of the matrix denotes the probability that a bidder submits the j-th lowest bid in the second round conditional on submitting the i-th lowest bid in the first round. When there are ties, multiple bidders are assigned to the same rank. The number of auctions is 8,016.

Table 2: Rank of the Second-Round Bid by Rank of the First-Round Bid

Group	(1) Partici-pation	(2) Rival Bidders	(3) Unique Rivals	(4) S_1	(5) S_2	(6) S_4	(7) S_{10}	(8) S_{15}	(9) Obs.
500+	807.8	8.5	434.4	62.7	57.5	47.3	28.9	17.9	34
	(299.4)	(1.3)	(266.1)	(18.3)	(17.2)	(15.0)	(8.7)	(7.2)	
100 - 500	176.8	9.2	294.1	40.8	36.4	30.7	20.3	14.6	658
	(80.7)	(0.7)	(175.6)	(19.6)	(18.9)	(16.5)	(9.3)	(5.6)	
50 - 100	69.0	9.3	140.6	49.9	44.3	37.4	24.8	18.2	1,130
	(13.6)	(0.9)	(66.4)	(18.6)	(17.2)	(15.2)	(9.1)	(5.7)	
15 - 50	28.5	9.2	77.7	57.3	50.8	42.9	28.3	20.4	3,426
	(9.8)	(1.1)	(35.6)	(18.8)	(18.0)	(16.1)	(9.5)	(7.5)	
1 - 15	4.3	9.0	23.6	85.4	79.8	71.7	44.7	36.5	17,566
	(3.7)	(2.0)	(15.7)	(19.8)	(22.7)	(25.4)	(22.3)	(24.4)	
Total	17.3	9.0	45.9	77.9	72.3	64.4	39.6	31.2	22,814
	(47.9)	(1.9)	(67.6)	(23.9)	(25.7)	(27.2)	(21.2)	(22.2)	

Table 3: Summary Statistics: Bidders.

Column (1) reports the average number of auctions in which a firm in a given group participates, Column (2) reports the average number of rival bidders in an auction, and Column (3) reports the cumulative number of unique opponents against whom a firm bids in the sample. For example, we find that the largest group of bidders participates in about

800 auctions and faces about 430 unique opponents, on average. Given that the average number of rival bidders in an auction is around 8.5, a firm could face a total of about 6,800 (=800×8.5) unique bidders if bidders never faced the same opponent. However, we find that the number of total unique bidders is a lot less, around 430. In Columns (4) - (8), we report additional measures that capture how often firms bid against each other. In particular, for each firm, we identify the rival who participates in the same auction with the firm most frequently, second most frequently, and so on. Column (4) reports the percentage share of auctions in which the most frequent opponent participates and Column (5) reports the percentage share in which the second most frequent opponent participates. Columns (6), (7), (8) correspond to the percentage shares for the fourth, tenth, and the fifteenth most frequent opponents. The fact that the numbers in Column (4) are higher than 40% for all rows means that, for the average firm, there exists a particular rival firm that is bidding on the same project more than 40% of the time.

4 Bidding Patterns in Reauctions

In this section, we document several bidding patterns that motivate our formal tests of competition. First, we document bidding patterns in reauctions that seem to suggest that losers of the initial auction can increase profits by bidding slightly more aggressively in reauctions. To the extent that this is actually the case, our finding rejects the null that bidders are maximizing expected profits. We formally test this hypothesis in Section 5.1. Second, we document what appears to be a sharp kink in the distribution of the bid difference between $i(1)$ and $i(2)$ in the second round and in the third round, where $i(k)$ denotes the identity of the bidder who submits the k-th lowest bid in Round 1. Lastly, we document differences between the set of bidders who marginally outbid $i(1)$ in the reauction and the set of bidders who are marginally outbid by $i(1)$. We find that bidders in the former set participate in auctions with lower winning bids, on average, than the latter. The second and the third bidding patterns that we document seem to reject the null that there exist factors that affect bids which are idiosyncratic to that bidder. We construct formal tests of competition based on the second and third patterns in Section 5.2

In what follows, we denote the (normalized) bid of bidder $i(k)$ in round t by $b^t_{i(k)}$. Because there is considerable variation in project size, we work with the normalized bids by dividing the actual bids by the reserve price of the auction. Hence, $b^2_{i(1)}$, for example, denotes the second-round bid of the first-round lowest bidder as a percentage of the reserve price.

Distribution of Second Round Bid Differences We begin with a plot of the histogram of $b^2_{i(2)} - b^2_{i(1)}$ ($= \Delta^2_{12}$), the second-round bid difference between $i(1)$ and $i(2)$. The top left panel of Figure 1 plots Δ^2_{12} for all auctions that reach the second round. The figure shows that Δ^2_{12} falls to the right of zero almost all of the time, which confirms what we report in Table 2: a flip in the rank order between the lowest and the second-lowest bidders almost never happens across rounds. We also find, however, that the margin by which $i(2)$ loses to $i(1)$ in the reauction is often small. This is reflected in the fact that the histogram of Δ^2_{12} has a substantial mass just to the right of zero. These findings suggest that $i(2)$ can increase the winning probability in the reauction substantially by bidding only slightly more aggressively.

Of course, the fact that $i(2)$'s winning probability in the reauction responds sharply to a small decrease in $b^2_{i(2)}$ does not immediately imply that $i(2)$'s current bid is suboptimal. If $i(2)$'s costs are relatively high, $i(2)$'s current bidding strategy may be optimal. The second through fourth panels of the left column of Figure 1, however, are suggestive that $i(2)$ has room to improve profits.

The second and third panels of the left column of Figure 1 plot Δ^2_{12} for auctions in which $i(1)$ and $i(2)$ bid relatively close to each other in the first round. In particular, the second panel conditions on $b^1_{i(2)} - b^1_{i(1)} < 5\%$ and the third panel conditions on $b^1_{i(2)} - b^1_{i(1)} < 1\%$. The fourth panel plots Δ^2_{12} conditional on the event that the three lowest bids in the first round are all within 1%, i.e., $b^1_{i(3)} - b^1_{i(1)} < 1\%$.[23]

Intuitively, bidders who lose to $i(1)$ by a relatively small margin in the initial round are likely to have similar costs as $i(1)$, on average, under competition. However, we find that the shape of the histogram of Δ^2_{12} remains similar from panel to panel. The figures suggest that bidders who have relatively low costs can also increase their winning probability by bidding slightly more aggressively. This, in turn, suggests that bidders who lose to $i(1)$ by a small margin in the initial auction are not bidding optimally. These findings motivate our test of competition in Section 5.1. There, we formally test for the optimality of second round bidding strategies of the losers in the initial auction. Our test is valid (i.e., has

[23] Note that conditioning on $b^1_{i(2)} - b^1_{i(1)} < 5\%$ and $b^1_{i(2)} - b^1_{i(1)} < 1\%$ still leaves us with relatively large sample sizes. For example, conditioning on 5% only reduces the sample of bids from 7,854 to 6,822. Conditioning on 1% leaves us with a sample size of 2,142. This is consistent with the findings in Chassang et. al. (2021) in which we document a distinct missing mass of almost tied bids while also finding a relatively large concentration of losing bids within a few percent of the lowest bid.

correct size) regardless of whether or not a bidder loses to $i(1)$ by a small or a large margin, although the test will turn out to have the most power for close losers in practice. The test takes into account the possible complications that arise from the fact that the lowest bid in each round are revealed to the other bidders.

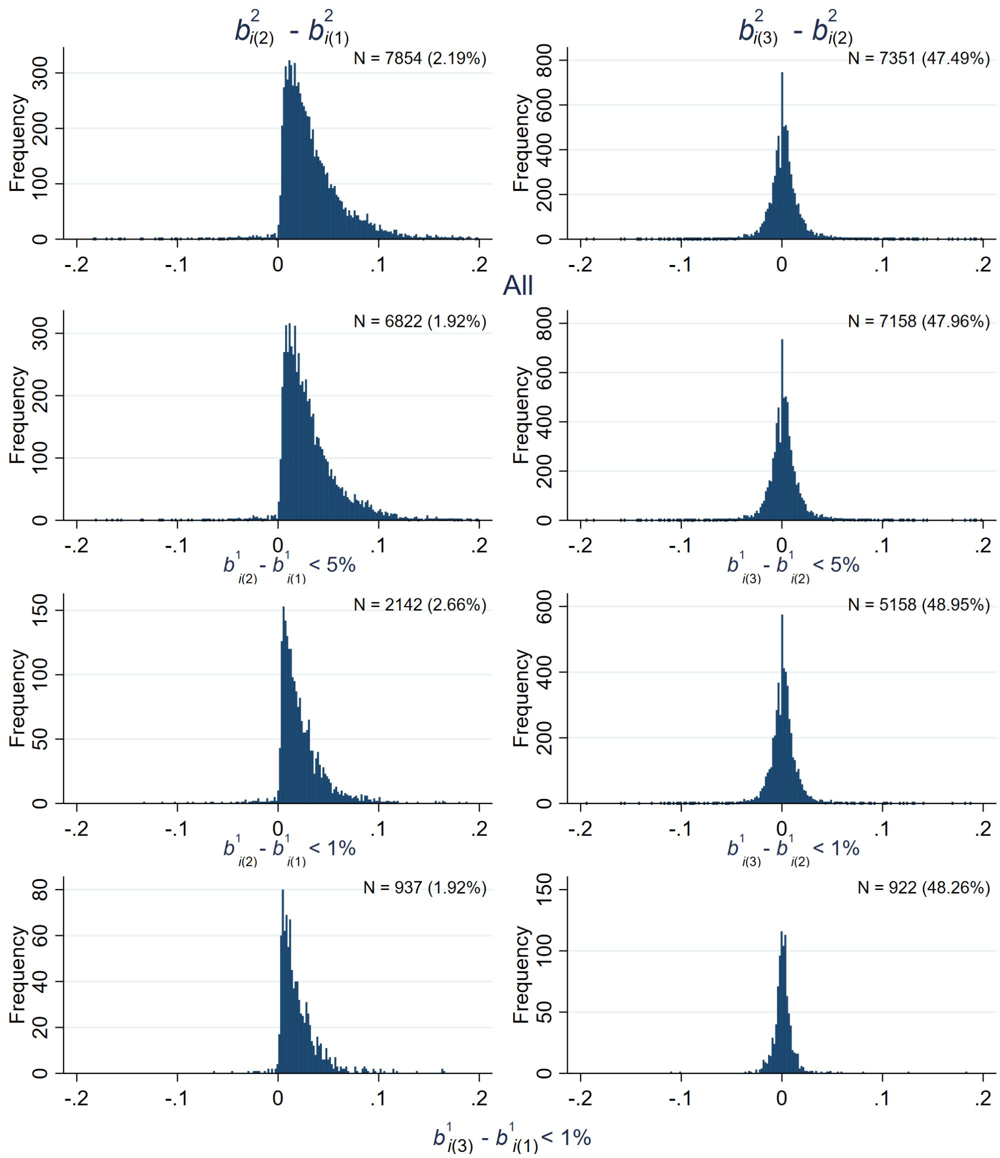
Another notable feature of the distribution of Δ_{12}^2 is what appears to be a kink at zero on the horizontal axis. We find that the number of observations in an interval to the left of zero, $[-t, 0]$ and the number of observations in an interval to the right of zero, $[0, t]$, are very different even for small values of $t > 0$. To the extent that there exists some idiosyncrasy among the bidders, $i(2)$ should outbid $i(1)$ in the second round by a narrow margin just as often as $i(1)$ outbids $i(2)$ by a narrow margin. That is, there should be a similar number of observations in which $\Delta_{12}^2 \in [-t, 0]$ and $\Delta_{12}^2 \in [0, t]$ for small values of t – a feature which we clearly do not see in any of the histograms of the left panels of Figure 2. This finding motivates our test in Section 5.2 which is based on the null that there exist idiosyncratic factors that affect bids if bidders are bidding competitively. A natural interpretation of these bidding patterns is that they are generated under a collusive scheme in which $i(1)$ is the designated bidder, other bidders know precisely how $i(1)$ is going to bid in the second round, and place bids above $i(1)$'s bid.[24]

For comparison, the right panels of Figure 1 plot the histogram of $b_{i(3)}^2 - b_{i(2)}^2$ $(= \Delta_{23}^2)$, the second-round bid difference between $i(2)$ and $i(3)$. Similar to the left panels, the top right panel is for all auctions that reach the second round, and the other panels correspond to conditioning on $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$, $b_{i(3)}^1 - b_{i(2)}^1 < 1\%$ and $b_{i(3)}^1 - b_{i(1)}^1 < 1\%$, respectively. In contrast to the left panels, we find that the shape of the histogram of Δ_{23}^2 is symmetric around zero, implying that the rank order between $i(2)$ and $i(3)$ flips in the second round with close to 50% probability.

In Online Appendix II, we plot the histograms of Δ_{12}^2 and Δ_{23}^2 conditional on various auction characteristics, such as region, project type, and year. We find that the distributions of Δ_{12}^2 and Δ_{23}^2 often look very similar to those shown in Figure 1: The distribution of Δ_{12}^2 is skewed to the right and displays what appears to be a discontinuity at $\Delta_{12}^2 = 0$, while the distribution of Δ_{23}^2 is symmetric around $\Delta_{23}^2 = 0$.[25] In Online Appendix II, we also

[24]Our findings suggest that bidding rings determine beforehand how each ring member should bid in the second round – not just how to bid in the first round. This is natural, given that a substantial fraction of auctions go to the second round and that there are only 30 minutes between rounds.

[25]In Online Appendix II, we also plot the differences in the homogenized bids. Homogenized bids are the residuals from a regression in which the bids are regressed on auction characteristics (See Haile et al., 2003). A plot of the differences of the homogenized bids also show very similar patterns.



The first row is the histogram for the set of auctions that reach Round 2, and $i(1)$ and $i(2)$ (or $i(2)$ and $i(3)$) submit valid bids in Round 2. The second to fourth rows plot the same histogram, but only for auctions in which the differences in the first-round bids are relatively small. N is the sample size, and the number in the parenthesis corresponds to the fraction of auctions that lie to the left of zero. The sample sizes are different between the top left and the top right panels because in some auctions, $i(1)$ or $i(3)$ does not bid in Round 2. Similarly for the bottom left and right panels.

Figure 1: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels).

plot Δ^2_{12} as well as Δ^2_{23} without normalizing the bids by the reserve price. The graphs also appear similar to Figure 1.[26]

Distribution of Third Round Bid Differences For the subset of auctions that reach the third round, we can further examine whether a similar bidding pattern that we find for the second round continues to hold in the third round. In the top panels of Figure 2, we plot the difference in the third-round bids of $i(1)$ and $i(2)$, i.e., $\Delta_{12}^3 \equiv b_{i(2)}^3 - b_{i(1)}^3$ (left panel), and the difference in the third-round bids of $i(2)$ and $i(3)$, i.e., $\Delta_{23}^3 \equiv b_{i(3)}^3 - b_{i(2)}^3$ (right panel) for all auctions that advance to the third round. In rows two to four of Figure 2, we plot the histogram conditioning on the set of auctions in which the first-round bids are relatively close. The conditioning sets are the same as those in Figure 1.

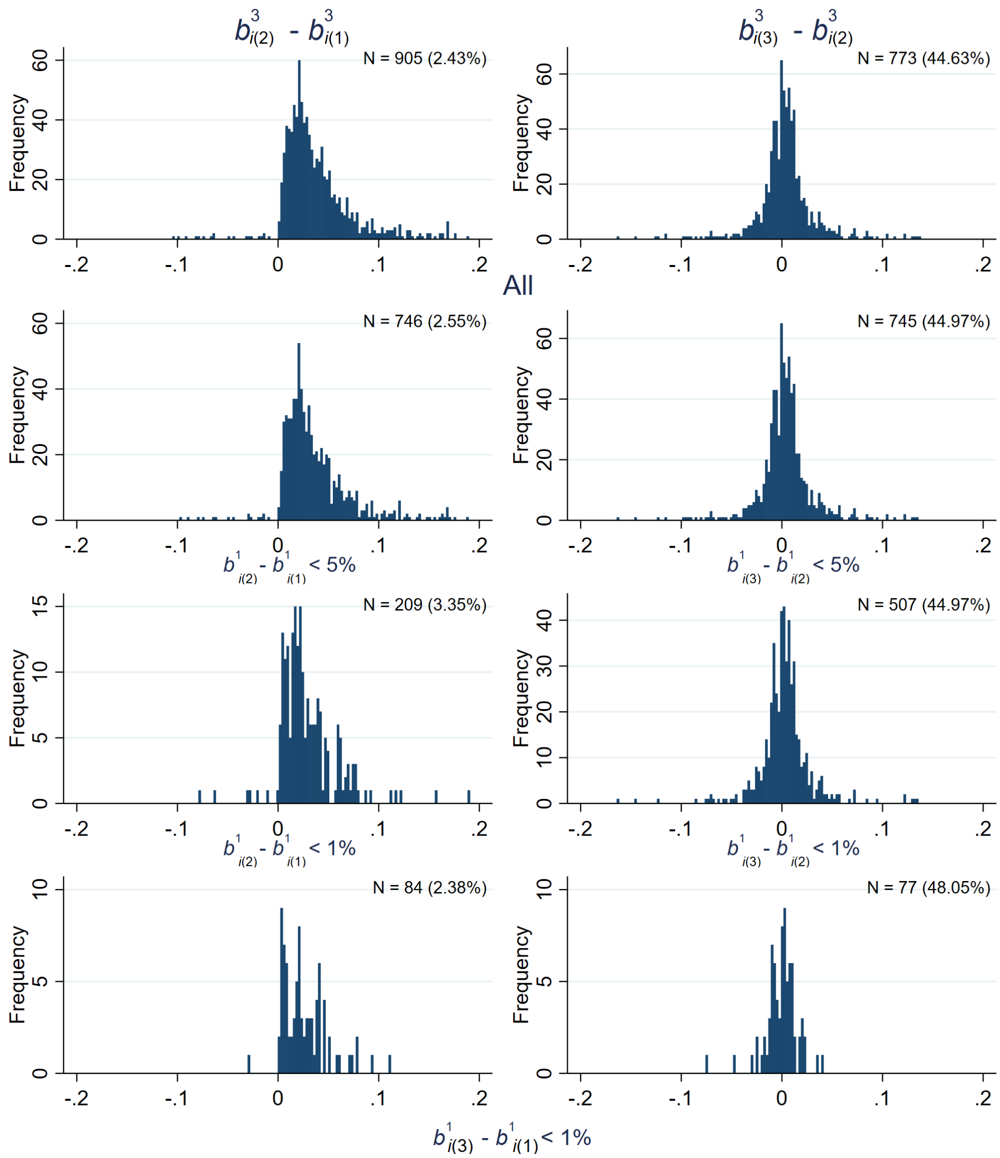
Overall, Figure 2 shows that bidding patterns in the third round are similar to those in the second round. We find that $i(1)$ almost always outbids $i(2)$, but often by a narrow margin. We also find that $i(3)$ outbids $i(2)$ about half of the time.

Outbidding $i(1)$ in the Second Round To the extent that collusive bidding implies that trailing bidders from the first round do not outbid $i(1)$ in the second round, an auction in which $i(1)$ is outbid in the second round is a sign of competition among bidders. Hence, if bidder i ($\neq i(1)$) outbids $i(1)$ in the second round, we expect i, on average, to be a more competitive bidder than those who do not outbid $i(1)$. We explore this idea by comparing the bidding behavior of firm i who outbids $i(1)$ in the second round (i.e., $b_i^2 < b_{i(1)}^2$) to those who do not (i.e., $b_i^2 > b_{i(1)}^2$). Specifically, for each auction n and each bidder $i \neq i(1)$, we consider the average winning bid of the five preceding and the five succeeding auctions in which bidder i participates. Ordering the auctions in which bidder i participates chronologically, and letting b_m^{win} denote the winning bid in auction m, we consider the following two statistics:

$$b_{i,n}^{before} = \frac{1}{5}\sum_{m=n-5}^{m=n-1} b_m^{win},\ b_{i,n}^{after} = \frac{1}{5}\sum_{m=n+1}^{m=n+5} b_m^{win}.$$

We compute $b_{i,n}^{before}$ and $b_{i,n}^{after}$ for each auction n that reaches the second round and for each bidder i ($\neq i(1)$). Note that we do not include the winning bid of auction n when

[26]In Online Appendix III, we also document bidding patterns from three municipal auctions. The format of the municipal auctions are almost exactly the same as the auctions in our sample with one difference, which is that none of the bids are announced at the end of each round in the municipal auctions. We find similar bidding patterns in reauctions for the municipal auctions as well.



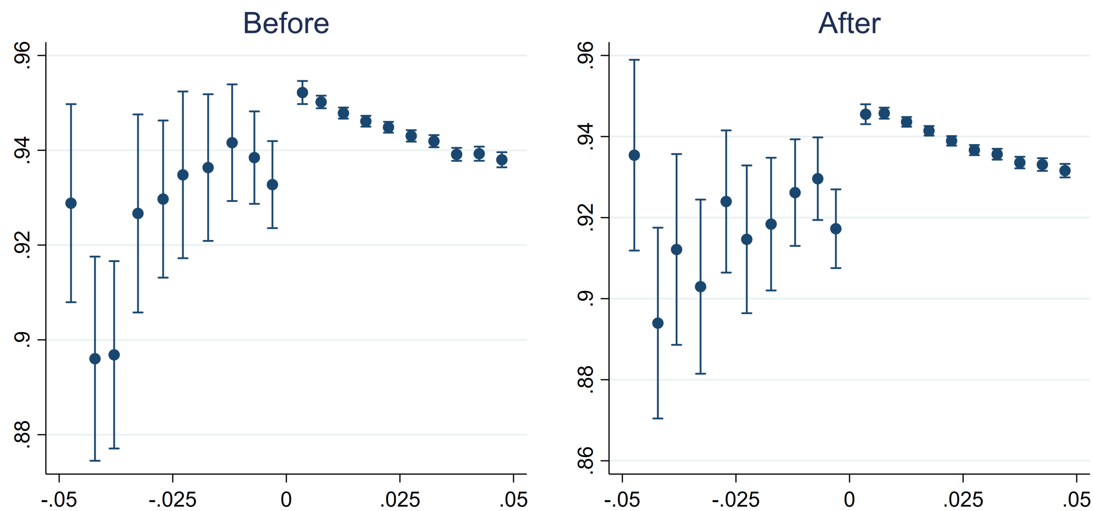
The first row corresponds to all auctions that reach the third round and $i(1)$ and $i(2)$ (in the case of the left panel) or $i(2)$ and $i(3)$ (in the case of the right panel) submit valid bids in the third round. The second to fourth rows plot the same histogram, but only for auctions in which the differences in the first-round bids are relatively small.

Figure 2: Difference in the Third-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Third-Round Bids of $i(2)$ and $i(3)$ (Right Panels).

computing $b^{before}_{i,n}$ and $b^{after}_{i,n}$

Figure 3 is a binned scatter plot of $b^{before}_{i,n}$ and $b^{after}_{i,n}$ against $b^2_{i,n} - b^2_{i(1),n}$.[27] In the left

[27] A more direct test may be to use the actual bids of bidder i to construct the test statistics (as opposed to the winning bids). With actual bids, we lose statistical significance at 5%.



The left panel plots $b_{i,n}^{before}$ against $b_{i,n}^2 - b_{i(1),n}^2$, and the right panel plots $b_{i,n}^{after}$ against $b_{i,n}^2 - b_{i(1),n}^2$. The bin size is 0.005. In both panels, the region to the left of zero corresponds to auctions in which bidder i ($\neq i(1)$) outbids $i(1)$, and the region to the right of zero corresponds to auctions in which $i(1)$ outbids bidder i.

Figure 3: Binned Scatter Plot of $b_{i,n}^{before}$ and $b_{i,n}^{after}$ Against $b_{i,n}^2 - b_{i(1),n}^2$.

panel of the figure, the vertical axis corresponds to $b_{i,n}^{before}$. The horizontal axis corresponds to $b_{i,n}^2 - b_{i(1),n}^2$, i.e., the difference in the second-round bids between $i(1)$ and i in auction n. The dots to the left of zero in the panel correspond to the bin averages of $b_{i,n}^{before}$ for bidders that trail in Round 1 and outbid $i(1)$ in Round 2. The dots to the right of zero correspond to $b_{i,n}^{before}$ for those that trail in Round 1 and lose to $i(1)$ in the second round. Vertical bars correspond to the confidence intervals. The sample size to the left of zero is much smaller (413) than the sample size to the right of zero (55,549). Hence, the bin averages are less precisely estimated to the left of zero. Nonetheless, the left panel shows that $b_{i,n}^{before}$ is lower, on average, to the left of zero than to the right of zero. The regression discontinuity estimate of the difference in $b_{i,n}^{before}$ at zero is 2.4 percentage points, and it is statistically significant at the 5% level.[28]

Similar to the finding that the distribution of Δ_{12}^2 has a kink at $\Delta_{12}^2 = 0$, the finding that the mean of $b_{i,n}^{before}$ is discontinuous (with respect to $b_{i,n}^2 - b_{i(1),n}^2$) at zero is inconsistent with the null that there exist idiosyncratic factors that affect bids under competitive bidding. To

[28]We use an estimator proposed in Calonico et al. (2018) with an Epanechnikov kernel and a mean square error optimal bandwidth.

the extent that there are idiosyncratic factors, the set of bidders just to the left of zero in Figure 3 should be similar to the set of bidders just to the right of zero, on average. In section 5.2, we formally show that this result violates our benchmark of competition.

The right panel of Figure 3 is a binned scatter plot of $b_{i,n}^{after}$ against $b_{i,n}^2 - b_{i(1),n}^2$. We find that the regression discontinuity estimate of $b_{i,n}^{after}$ at $b_{i,n}^2 - b_{i(1),n}^2 = 0$ is 2.7 percentage points, and statistically significant at the 95% level. Although this panel is also consistent with the notion that persistence in the identity of the lowest bidder is symptomatic of collusion, it is not as clean as the left panel. This is because the right panel plots the winning bid of auctions that take place after the auction in question. A bidder who outbids $i(1)$ is likely to be the winner of the auction, which introduces asymmetry between a bidder who marginally loses to $i(1)$ and a bidder who marginally defeats $i(1)$, from that point onward. To the extent that backlog affects future bidding behavior, the difference at zero may capture that effect.

5 Formal Tests of Competitive Bidding

In this section, we propose formal tests of competitive bidding and apply them to our data. We consider two benchmarks of competitive behavior. The first is expected profit maximization with rational expectations and the second is existence of idiosyncratic factors that affect bids. These benchmarks reflect the notion that, under competition, bidders bid optimally to first-order approximation, but that small optimization errors and idiosyncratic shocks are also prevalent. In order to test the former, we test for the optimality of the bidding strategy in the reauction employed by bidders who lose in the initial auction. In order to test for the latter, we test whether or not the marginal winners of the second round are similar to marginal losers of the second round.

5.1 Optimality of Second-Round Bidding Strategy

Recall that there are many cases in which $i(2)$ could have outbid $i(1)$ in the second round by lowering its second-round bid by a tiny margin. For example, focusing on the left panel of the second row in Figure 1, we find that about 15.75% and 38.67% of the distribution of Δ_{12}^2 lie within $[0,\ 0.01]$ and $[0,\ 0.02]$, respectively. On the other hand, the fraction of the distribution that lies to the left of zero is only 1.73%. This suggests that $i(2)$ can increase the probability of outbidding $i(1)$ substantially by decreasing its bid only slightly, raising the question of whether $i(2)$'s second-round bid is optimal.[29]

Based on this observation, we construct a formal test of competition taking the null of competition as expected profit maximization in the one-shot (i.e., not repeated) game, where the one-shot game includes the initial auction and the subsequent reauctions for the same project but not lettings for other projects. In particular, we test whether or not the second-round bidding strategy employed by bidders who lose in the first round is optimal. Note that expected profit maximization is a necessary condition of Bayes Nash equilibria of the one-shot game.[30] Rejection of expected profit maximization is suggestive of collusive repeated-game incentives that keep the bidders from bidding optimally in the one-shot game.

The key idea behind the test is that the firm's third-round bid can provide an upper bound on its costs under private values. Using this idea, we can compute a lower bound on the bidder's profits from playing an alternative bidding strategy in the second round without fully characterizing the equilibrium. This approach is similar in spirit to that of Haile and Tamer (2003), who obtain an upper bound on the value of bidders in an incomplete model of English auctions using the assumption that bidders do not bid above their value.

In what follows, we compare, for bidders that lose in the first round, the expected profits from using the current second-round strategy and the expected profits from using alternative second-round strategies. The alternative strategies that we consider are of the form $x\mathfrak{b}_i^2$, where x is some number less than 1 (e.g., 0.99) and $\mathfrak{b}_i^2$ is the bidder's current (unnormalized) second-round bidding strategy. Just for this section, we work with the raw bids without normalizing by the reserve price. We show below that, for a range of values of x, the expected profits actually increase.

First, let i be a bidder who bid higher than $i(1)$ in the first round and $\mathfrak{b}_i^2$ be its current bidding strategy in the second round. Note that the strategy (which can be a mixed strategy) depends on the information revealed to bidder i in the first round, denoted as $\mathcal{J}$, which includes its own costs, c_i, its own bid, $\mathfrak{b}_i^1$, the lowest bid, $\min_j \mathfrak{b}_j^1$, and the fact that the secret reserve, r, is less than the lowest bid (in addition to observable auction characteristics):

$$\mathcal{J} = (c_i, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \{r < \min_j \mathfrak{b}_j^1\}).$$

[29]Strictly speaking, $i(2)$ does not know that it came in second at the time of rebidding (it learns only that it came close to being first). The analysis below takes this into consideration.

[30]In Online Appendix VI, we characterize a mixed-strategy equilibrium with two rounds and two bidder types.

The expected profits of bidder i consist of two components: the expected profits from winning in the second round; and the expected profits from being the lowest bidder in the third round if the auction advances to the third round. We denote by W^2 the event that bidder i wins in the second round and by W^3 the event that bidder i is the lowest third-round bidder,

$$\begin{aligned} W^2 &= \{\mathfrak{b}_i^2 < \min\{r, \min_{j \neq i} \mathfrak{b}_j^2\}\} \\ W^3 &= \{\mathfrak{b}_i^3 < \min_{j \neq i} \mathfrak{b}_j^3 \text{ and } \min_j \mathfrak{b}_j^2 > r\}, \end{aligned}$$

where $\mathfrak{b}_i^3$ is bidder i's current third-round bidding strategy.[31] We now express bidder i's expected profits under $\mathfrak{b}_i^2$:

$$\pi_{i|\mathcal{J}} = \Pr(W^2|\mathcal{J})\mathbf{E}_{\mathcal{J}}[\mathfrak{b}_i^2 - c_i|W^2] + \Pr\left(W^3|\mathcal{J}\right)\mathbf{E}_{\mathcal{J}}[\text{ profits }|W^3].$$

The profits in event W^3 is either $\mathfrak{b}_i^3 - c_i$ if $\mathfrak{b}_i^3$ is lower than r, or some number less than $\mathfrak{b}_i^3 - c_i$ (which depends on how the bilateral negotiation between bidder i and the government plays out) if $\mathfrak{b}_i^3$ is higher than r. In either case, the expected profits in event W^3 are less than $\mathbf{E}_{\mathcal{J}}[\mathfrak{b}_i^3|W^3]$. Thus, we can bound $\pi_{i|\mathcal{J}}$ from above as follows:

$$\pi_{i|\mathcal{J}} \leqq \Pr(W^2|\mathcal{J})\mathbf{E}_{\mathcal{J}}[\mathfrak{b}_i^2 - c_i|W^2] + \Pr\left(W^3|\mathcal{J}\right)\mathbf{E}_{\mathcal{J}}[\mathfrak{b}_i^3|W^3].$$

Now consider the expected profits, $\tilde{\pi}_{i|\mathcal{J}}$, from an alternative second-round bidding strategy that discounts current second-round bids by some factor $x \in (0,1)$. As before, $\tilde{\pi}_{i|\mathcal{J}}$ consists of two components, the expected profits from the second round and the expected profits from the third round:

$$\tilde{\pi}_{i|\mathcal{J}} = \Pr(\tilde{W}^2|\mathcal{J})\mathbf{E}_{\mathcal{J}}[x\mathfrak{b}_i^2 - c_i|\tilde{W}^2] + \mathbf{E}_{\mathcal{J}}[\text{third round profits}],$$

where $\tilde{W}^2$ is the event in which bidder i wins in the second round using strategy $x\mathfrak{b}_i^2$, i.e., $\{x\mathfrak{b}_i^2 < \min\{r, \min_{j \neq i} \mathfrak{b}_j^2\}\}$. Because we are interested only in obtaining a lower bound for $\tilde{\pi}_{i|\mathcal{J}}$, it is not necessary to specify $\mathbf{E}_{\mathcal{J}}[\text{third-round profits}]$ other than to note that it is

[31] $\mathfrak{b}_i^3$ depends on the information available to bidder i after two rounds of bidding, i.e., $(c_i, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \{r < \min_j \mathfrak{b}_j^1\}, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, \{r < \min_j \mathfrak{b}_j^2\})$.

nonnegative. Thus, we obtain a lower bound on $\tilde{\pi}_{i|\mathcal{J}}$ as follows:

$$\tilde{\pi}_{i|\mathcal{J}} \geqq \Pr(\tilde{W}^2|\mathcal{J})\mathbf{E}_{\mathcal{J}}[x\mathfrak{b}_i^2 - c_i|\tilde{W}^2]. \tag{1}$$

We now compare the change in expected profits, $\Delta\pi_{i|\mathcal{J}}$, from bidding $x\mathfrak{b}_i^2$ instead of $\mathfrak{b}_i^2$. Using the bounds obtained above, $\Delta\pi_{i|\mathcal{J}}$ can be bounded below as follows:

$$\begin{aligned}\Delta\pi_{i|\mathcal{J}} \equiv \tilde{\pi}_{i|\mathcal{J}} - \pi_{i|\mathcal{J}} &\geqq \Pr(\tilde{W}^2 - W^2|\mathcal{J})\mathbf{E}_{\mathcal{J}}[x\mathfrak{b}_i^2 - c_i|\tilde{W}^2 - W^2] \\ &\quad - \Pr(W^2|\mathcal{J})\mathbf{E}_{\mathcal{J}}[(1-x)\mathfrak{b}_i^2|W^2] - \Pr(W^3|\mathcal{J})\mathbf{E}_{\mathcal{J}}[\mathfrak{b}_i^3|W^3], \end{aligned} \tag{2}$$

where $\tilde{W}^2 - W^2 = \tilde{W}^2 \cap (W^2)^C$. Note that $\tilde{W}^2 - W^2$ is the event in which bidder i wins in the second round with $x\mathfrak{b}_i^2$ but not with $\mathfrak{b}_i^2$. Because we consider $x \in (0,1)$, $\tilde{W}^2$ is a superset of W^2, i.e., $\tilde{W}^2 \supset W^2$. The potential gain from using strategy $x\mathfrak{b}_i^2$ instead of $\mathfrak{b}_i^2$ occurs in event $\tilde{W}^2 - W^2$, and the amount of the gain is $(x\mathfrak{b}_i^2 - c_i)$.[32] The first term on the right-hand side of expression (2) corresponds to the gain. The second term corresponds to the potential loss from using $x\mathfrak{b}_i^2$. In event W^2, using $x\mathfrak{b}_i^2$ is less profitable than $\mathfrak{b}_i^2$ because bidder i is already winning with a bid of $\mathfrak{b}_i^2$. Note that a necessary condition of expected profit maximization is that firm i has no profitable deviation, i.e., $\Delta\pi_{i|\mathcal{J}}$ is nonpositive for each $\mathcal{J}$.

In order to derive conditions that we can take to the data, consider $\mathcal{H}$, which is a coarser partition of $\mathcal{J}$:

$$\mathcal{H} = (\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \{r < \min_j \mathfrak{b}_j^1\})$$

The difference between $\mathcal{J}$ and $\mathcal{H}$ is that $\mathcal{J}$ includes c_i but $\mathcal{H}$ does not. Taking expectations of expression (2) with respect to $\mathcal{H}$, we obtain the following expression:

$$\begin{aligned}\Delta\pi_{i|\mathcal{H}} &\equiv \mathbf{E}_{\mathcal{H}}\left[\Delta\pi_{i|\mathcal{J}}\right] \\ &\geqq \Pr(\tilde{W}^2 - W^2|\mathcal{H})\mathbf{E}_{\mathcal{H}}[x\mathfrak{b}_i^2 - c_i|\tilde{W}^2 - W^2] \\ &\quad - \Pr(W^2|\mathcal{H})\mathbf{E}_{\mathcal{H}}[(1-x)\mathfrak{b}_i^2|W^2] - \Pr(W^3|\mathcal{H})\mathbf{E}_{\mathcal{H}}[\mathfrak{b}_i^3|W^3]. \end{aligned} \tag{3}$$

Given that expected profit maximization requires $\Delta\pi_{i|\mathcal{J}}$ to be nonpositive for each $\mathcal{J}$, it also requires $\Delta\pi_{i|\mathcal{H}}$ to be nonpositive for each $\mathcal{H}$.

Taking expectations with respect to $\mathcal{H}$ allows us to get closer to expressions that we

[32] If c_i is higher than $x\mathfrak{b}_i^2$, this will not be a gain, but a loss.

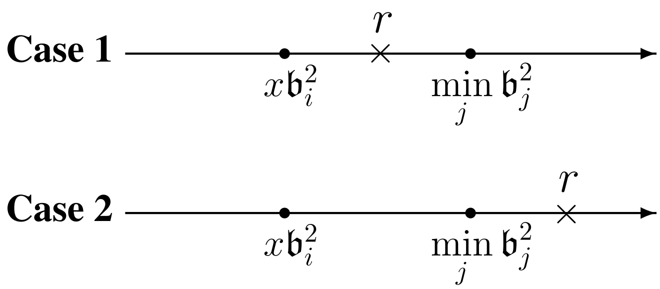


Figure 4: Event $\tilde{W}^2 - W^2$ includes two possibilities, one in which r happens to be below the lowest bid in the second round (Case 1) and the other in which r happens to be above the lowest bid in the second round (Case 2).

can take to the data. All of the terms in expression (3), except for $\mathbf{E}_{\mathcal{H}}[c_i|\tilde{W}^2 - W^2]$, can be evaluated directly from the data, in the sense that sample analogues can be constructed (assuming that $\mathcal{H}$ does not include characteristics that are unobservable to the econometrician). For example, for any given value x, $\mathbf{E}_{\mathcal{H}}[x\mathfrak{b}_i^2|\tilde{W}^2 - W^2]$ can be evaluated by taking the sample average of $x\mathfrak{b}_i^2$ for auctions in which (1) bidder i bids $\mathfrak{b}_i^1$ in the first round, (2) the lowest first-round bid is $\min_j \mathfrak{b}_j^1$, and (3) a bidder does not win in the second round, but would have won if it had bid x (e.g., 0.99) of the original bid.[33] The only term that we cannot evaluate directly is $\mathbf{E}_{\mathcal{H}}[c_i|\tilde{W}^2 - W^2]$ because we do not know c_i. However, under the private values assumption, it turns out that we can bound this term using the bidder's third-round bid. We discuss this issue next.

Recall that $\tilde{W}^2 - W^2$ corresponds to the event in which bidder i wins in the second round with $x\mathfrak{b}_i^2$ but not with $\mathfrak{b}_i^2$. Event $\tilde{W}^2 - W^2$ includes two possibilities, one in which r happens to be below the lowest bid in the second round ($\{r < \min_j \mathfrak{b}_j^2\}$) and the other in which r happens to be above ($\{r \geqq \min_j \mathfrak{b}_j^2\}$). Figure 4 depicts the two situations. Note that for Case 1, the auction proceeds to the third round, and we observe $\mathfrak{b}_i^3$. Hence, we can bound c_i from above by the observed third-round bid, $\mathfrak{b}_i^3$.[34] For Case 2, however, the auction ends in the second round, and we do not observe third-round bids.

We now consider how to put bounds on c_i for Case 2. Note that whether or not the auction proceeds to the third round is, to some extent, independent of the bidders' costs. It depends, in part, on the random realization of r. The lemma below makes this statement

[33]Note that conditioning on b_i^1 and $\min_j b_j^1$ is necessary because we need to condition on $\mathcal{H}$.

[34]Even for Case 1, we do not observe the third-round bid when the bidder decides not to bid in the third round. When this is the case, we use the bidder's second round bid as an upper bound, which is a very conservative bound.

precise; it states that if we have two auctions with the same realizations of $\{\mathfrak{b}_i^1$, $\min_j \mathfrak{b}_j^1$, $\mathfrak{b}_i^2$, $\min_j \mathfrak{b}_j^2\}$, but one ending in the second round and the other proceeding to the third round, bidder i's costs must be the same, on average, in the two auctions. This lemma generalizes the observation that, if bidder i plays a pure monotone strategy, two auctions with the same realizations of $\mathfrak{b}_i^1$ implies the same realization of c_i.[35] The lemma allows us to bound bidder i's costs for Case 2 by using the third-round bids conditional on $\mathcal{H}$ and $\{\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2\}$.

Lemma *Assume that bidders have private costs* $\mathbf{c} = (c_1, \cdots, c_N)$*; that* $\mathbf{c}$ *has density; and that* $\mathbf{c} \perp r$*. Then,*

$$\begin{aligned} &\mathbf{E}_{\mathcal{H}}[c_i | \{r \geqq \min_j \mathfrak{b}_j^2\}, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2] \\ &= \mathbf{E}_{\mathcal{H}}[c_i | \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2] \\ &= \mathbf{E}_{\mathcal{H}}[c_i | \{r < \min_j \mathfrak{b}_j^2\}, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2]. \end{aligned} \tag{4}$$

Moreover,

$$\begin{aligned} &\mathbf{E}_{\mathcal{H}}[c_i | (\tilde{W}^2 - W^2) \cap \{r \geqq \min_j \mathfrak{b}_j^2\}] \\ &\qquad \leqq \mathbf{E}_{\mathcal{H}}[h_{\mathcal{H}}(\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) | (\tilde{W}^2 - W^2) \cap \{r \geqq \min_j \mathfrak{b}_j^2\}], \end{aligned} \tag{5}$$

where $h_{\mathcal{H}}(\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) = \mathbf{E}_{\mathcal{H}}[\mathfrak{b}_i^3 | \{r < \min_j \mathfrak{b}_j^2\}, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2]$.

Proof. See the Appendix. ■

The first part of the lemma states that, conditional on $\{\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2\}$, the expected cost of bidder i for an auction that ends in the second round given $\mathcal{H}$ (the first line of expression (4)) is the same as the expected cost of bidder i for an auction that goes to the third round given $\mathcal{H}$ (the third line of expression (4)) – under the assumption of private values and $\mathbf{c} \perp r$. We argue below that these two assumptions are relatively innocuous in our setting. The second part of the lemma states that we can bound c_i in Case 2 using the mean of the observed third-round bids. Note that the left-hand side of inequality (5) is the expected bidder cost conditional on Case 2. This is bounded by the conditional expectation of $h_{\mathcal{H}}(\cdot)$, which is the expectation of $\mathfrak{b}_i^3$ conditional on $\{\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, \{r < \min_j \mathfrak{b}_j^2\}\}$.

[35] If bidder i plays a pure monotone strategy, $\mathfrak{b}_i^1$ is fully revealing about i's costs. In particular, $\mathbf{E}_{\mathcal{H}}[c_i|\{r < \min_j \mathfrak{b}_j^2\}] = \mathbf{E}_{\mathcal{H}}[c_i|\{r \geqq \min_j \mathfrak{b}_j^2\}]$ given that information set $\mathcal{H}$ includes $\mathfrak{b}_i^1$. Because we allow for mixed strategies, we need to condition on $\{\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2\}$.

The reason that the Lemma is useful is that the right-hand side of expression (5) can be computed using observed data. In particular, $h_{\mathcal{H}}(\cdot)$ can be estimated by the sample mean of the observed third-round bids conditional on $\{\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2\}$ and $\mathcal{H}$. In practice, our estimate of $h_{\mathcal{H}}(\cdot)$ is a linear projection of $\mathfrak{b}_i^3$ on $\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2$ as well as auction characteristics such as year, region, and project types, based on the subset of auctions that reach the third round. We can then use the estimated function, $h_{\mathcal{H}}(\cdot)$, to predict what the value of $\mathfrak{b}_i^3$ would have been for Case 2 as $h_{\mathcal{H}}(\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2)$.[36] The average of $h_{\mathcal{H}}(\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2)$ among all Case 2 auctions conditional on $\mathcal{H}$ corresponds to the right-hand side of expression (5).

Before we present our results, we briefly discuss the two assumptions of the Lemma, namely, that bidders have private values and that the costs and the reserve price are independent. We start with the private values assumption. By assuming that bidders have private values, c_i becomes constant throughout the three rounds, ensuring that $\mathfrak{b}_i^3$ is a valid upper bound for the costs of bidder i perceived at the time of the second round. Note that the private values assumption is sufficient but not necessary for $\mathfrak{b}_i^3$ to be an upper bound for c_i at Round 2. If, instead, bidders have common values, bidders may update their costs in the third round based on the observed lowest second-round bid. Our results hold as long as $\mathfrak{b}_i^3$ can be used as an upper bound on the costs of bidder i perceived at the time of the second round.[37]

Next, we discuss the independence of $\mathbf{c}$ and r.[38] One might be inclined to argue that the independence assumption is violated based on, for example, the observation that $\mathbf{c}$ and r are both low for simple jobs (e.g., road paving) and that they are both high for complicated jobs (e.g., bridges). This is *not* necessarily a valid criticism of the independence assumption. In this example, all of the players should be aware that there are two completely different sets of distributions from which $\mathbf{c}$ and r are drawn, one for road paving and the other for bridges. It is not the case that there is one common set of distributions of $\mathbf{c}$ and r for both paving and bridges. Conditional on what is common knowledge to the players, $\mathbf{c}$ and r may very well be independent even in this example.[39] As long as $\mathbf{c}$ and r are independent

[36] Intuitively, for each auction that ends in the second round, we can find another auction that reaches the third round and has the same realization of $(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2)$ as the former. The third-round bids of the latter can be used as a bound for c_i in the former.

[37] If bidders revise their costs up conditional on reaching the third round, our results would continue to hold. Online Appendix VII analyzes the sensitivity of our results to downward revision of costs.

[38] We do not need bidder costs $\{c_i\}$ to be independent or identical.

[39] For example, suppose that bidder costs and the reserve price for paving are given by $c_i^p = \mu^p + \epsilon_i^p$ and $r^p = \mu^p + \epsilon^p$, and similarly for bridges, $c_i^b = \mu^b + \epsilon_i^b$ and $r^b = \mu^b + \epsilon^b$. Suppose, also, that μ^p and μ^b are constants commonly observed by the bidders, whereas ϵ_i^p and ϵ_i^b are privately known to the bidders. Then, $\mathbf{c}$ and r are independent from the perspective of the bidders as long as $\epsilon_i^p \perp \epsilon^p$ and $\epsilon_i^b \perp \epsilon^b$ (although they are

conditional on observable characteristics, the results of the Lemma remain true.

The reason why we think that conditional independence is not a bad assumption is because of the way the reserve price is constructed. As we described in Section 2, there is a formula for converting various procedures into input quantities. The reserve price is essentially the sum of the inputs multiplied by their unit price. However, there is rounding in the process of computing the reserve price, which make the reserve price random from the perspective of bidders. To the extent that the randomness in the reserve price stems from the way rounding is applied in each step, the reserve price is plausibly independent of private cost realizations at the firm level.

We are now ready to evaluate $\Delta\pi_{i|\mathcal{H}}$, the difference in the expected profits from using $x\mathfrak{b}_i^2$ instead of $\mathfrak{b}_i^2$ in the second round. Using expressions (2) and (5) and the fact that $\mathfrak{b}_i^3 > c_i$ for Case 1, we obtain the following bound:

$$\Delta\pi_{i|\mathcal{H}} \geqq \underline{\Delta\pi_{i|\mathcal{H}}}, \tag{6}$$

where

$$\begin{aligned}
\underline{\Delta\pi_{i|\mathcal{H}}} &= \Pr(\tilde{W}^2 - W^2|\mathcal{H})\mathbf{E}_{\mathcal{H}}[x\mathfrak{b}_i^2|\tilde{W}^2 - W^2] - \Pr(W^2|\mathcal{H})\mathbf{E}_{\mathcal{H}}[(1-x)\mathfrak{b}_i^2|W^2] \\
&\quad - \Pr(W^3|\mathcal{H})\mathbf{E}_{\mathcal{H}}[\mathfrak{b}_i^3|W^3] - \Pr(\{\text{Case 1}\}|\mathcal{H})\mathbf{E}_{\mathcal{H}}[\mathfrak{b}_i^3|\{\text{Case 1}\}] \\
&\quad - \Pr(\{\text{Case 2}\}|\mathcal{H})\mathbf{E}_{\mathcal{H}}[h_{\mathcal{H}}(\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2)|\{\text{Case 2}\}]; \\
\{\text{Case 1}\} &= (\tilde{W}^2 - W^2) \cap \{r < \min_j \mathfrak{b}_j^2\}; \text{ and} \\
\{\text{Case 2}\} &= (\tilde{W}^2 - W^2) \cap \{r \geqq \min_j \mathfrak{b}_j^2\}.
\end{aligned}$$

As explained earlier, we can construct sample analogues of all of the terms on the right-hand side of expression (6) using all auctions that reach the second round. For auctions that end in the second round, we use the predicted third round bid, $h_{\mathcal{H}}(\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2)$ as a bound on costs, and for auctions that reach the third round, we use the realized third round bid, $\mathfrak{b}_i^3$.

Although it is possible to use inequality (6) directly to construct our test, given that evaluating the inequality for each $\mathcal{H}$ requires a lot of data, we work with an inequality that pools across $\mathcal{H}$. Note that, for any coarser partition, $\mathcal{I} \subset \mathcal{H}$, we can construct an inequality analogous to expression (6) in which $\mathcal{H}$ is replaced with $\mathcal{I}$ (except for the subscript in

correlated unconditionally if $\mu^p \neq \mu^b$).

$h_{\mathcal{H}}$).[40] Hence, the null hypothesis that we take to the data is as follows:

$$H_0\colon 0 \geqq \underline{\Delta\pi_{i|\mathcal{I}}}. \tag{7}$$

We report our estimates of $\underline{\Delta\pi_{i|\mathcal{I}}}$ for $\mathcal{I} = \{\mathfrak{b}_i^1 - \min_j \mathfrak{b}_j^1 < \delta \min_j \mathfrak{b}_j^1\} \cap \{r < \min_j \mathfrak{b}_j^1\}$ with four different values of δ (1%, 3%, 5%, 15%) and five different values of x (99%, 98.5%, 98%, 97.5%, 97%) in Table 4.[41] Here, $\mathcal{I}$ also pools across auction characteristics.[42] Note that $\{\mathfrak{b}_i^1 - \min_j \mathfrak{b}_j^1 < \delta \min_j \mathfrak{b}_j^1\}$ corresponds to the event that a bidder loses to the lowest bidder by less than δ in the first round. Thus, each cell in Table 4 represents the lower bound of the change in expected profits from using $x\mathfrak{b}^2$ instead of $\mathfrak{b}^2$ for all bidders who lose in the first round by less than δ.[43]

Note that all of the cells in Table 4 are positive, implying that firms would be able to increase expected profits by decreasing their second-round bids by a small margin. We reject H_0 with a 5% significance level for all combinations of (x, δ). In terms of magnitude, the numbers seem quite large, considering how loose our inequality is. For example, looking at $(x, \delta) = (98\%, 1\%)$, we see that the bidder can increase its expected profits by around 1.7 million yen, on average, by decreasing its second-round bids by 2%. Relative to the mean reserve price of about 83 million yen for auctions that proceed to the second round, this seems substantial. Our results suggest that bidders are not bidding competitively.

Bounding Damages from Collusion We now consider using the expected profit gains computed in Table 4 to put bounds on damages from collusion. Because we compute expected profit gains by simply comparing profits from using current strategies with those from using alternative strategies, our estimates of foregone profits are also consistent estimates of how much cartel members stand to gain by deviation *under the null of collusion*.[44]

[40] We can take expectations of equation (6) with respect to $\mathcal{I}$ to obtain $\underline{\Delta\pi_{i|\mathcal{I}}} = \mathbf{E}_{\mathcal{I}}[\underline{\Delta\pi_{i|\mathcal{H}}}]$. $\underline{\Delta\pi_{i|\mathcal{I}}}$ is equal to the expression in which $\Pr(\cdot|\mathcal{H})$ is replaced by $\Pr(\cdot|\mathcal{I})$ and $\mathbf{E}_{\mathcal{H}}$ is replaced by $\mathbf{E}_{\mathcal{I}}$ in the expression for $\underline{\Delta\pi_{i|\mathcal{H}}}$.

[41] We use bootstrap to compute our standard errors. Note that we can estimate $\underline{\Delta\pi_{i|\mathcal{I}}}$ as a semiparametric M-estimator with one finite parameter of interest, $\underline{\Delta\pi_{i|\mathcal{I}}}$, and an infinite dimensional nuisance parameter $h_{\mathcal{H}}(\cdot)$. Cheng and Huang (2010) show that bootstrap confidence intervals for M-estimators yield asymptotically correct coverage probability. We estimate $h(\cdot)$ using a linear regression of $(\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1)$ as well as a region, year and project fixed effects. The confidence intervals are based on 100 bootstrap estimates.

[42] We have implicitly assumed that both $\mathcal{J}$ and $\mathcal{H}$ include observable auction characteristics.

[43] Note that we do not need δ to be small for the tests to be valid. The validity of the tests do not depend on whether the bidder loses to $i(1)$ by a small or a large margin.

[44] The amount of money that cartel members are willing to forego today in order to maintain collusion is one way of gauging the sustainability of cartels. If bidders are willing to forego large amounts of money

	$\Delta\pi_{i\|\mathcal{I}}$ (Yen)					
	$x = 99.0\%$	98.5%	98.0%	97.5%	97.0%	N
$\delta = 1\%$	1,012,278	1,542,406	1,747,725	1,761,603	1,667,326	4,499
	(173,089)	(266,948)	(318,516)	(354,170)	(384,993)	
3%	554,285	939,417	1,205,290	1,338,467	1,328,077	26,008
	(106,318)	(155,842)	(199,131)	(237,467)	(264,995)	
5%	414,559	732,801	978,444	1,115,560	1,123,030	42,141
	(76,027)	(117,492)	(155,527)	(188,222)	(212,365)	
15%	293,715	532,802	723,329	841,389	856,455	66,124
	(52,882)	(83,946)	(112,661)	(137,651)	(155,762)	

Note: Bootstrap standard errors are reported in parentheses. All the numbers are in Yen.

Table 4: Expected Gain in Profits from Bidding $x\mathfrak{b}_i^2$.

Given that, under the standard repeated-game model of collusion, deviation profits must be lower than the difference between the continuation value of collusion and that of defection, our estimates of deviation profits can be used to bound the differences in the continuation values of cartel bidders. We then use the fact that the differences in continuation values are decomposed into a price effect and an efficiency effect to estimate bounds on the damages of collusion under the null that bidders are colluding.

In order to implement these ideas, consider a standard repeated-game model of a bidding ring in which bidders take turns winning. The key incentive compatibility constraint implied by subgame perfection is as follows:

$$\text{[Deviation Profit]} \leq V_i^{\text{Coll}} - V_i^D, \tag{8}$$

where the left-hand side of the inequality is today's gain by deviating from prescribed play and the right-hand side of the inequality is the difference between the continuation value of colluding (V_i^{Coll}) and that of deviating (V_i^D). Note that our estimates in Table 4 are consistent lower-bound estimates of deviation profits under the null of collusion (under the assumption of private values, $r \perp \mathbf{c}$, and that bids are above costs). Assuming that the cartel plays static Nash after a bidder defects,[45] the difference in continuation values can

today to maintain collusion, it suggests that the cartel is that much more resilient.

[45]This is not an innocuous assumption. In general, the optimal punishment is not static Nash reversion. On the other hand, renegotiation may be an important consideration. See, e.g., Farrell and Maskin (1989).

be expressed in terms of the differences in profits under collusion and competition:

$$V_i^{\text{Coll}} - V_i^D = \mathbb{E}\sum_t [\beta(1-\delta)]^t \{\Pr(i \text{ wins at } t|\text{Coll})(\mathfrak{b}_{i,t}^{\text{Coll}} - c_{i,t}) - \Pr(i \text{ wins at } t|\text{NE})(\mathfrak{b}_{i,t}^{\text{NE}} - c_{i,t})\},$$

where β is the discount factor, δ is the probability of collusion breaking down, $\Pr(i \text{ wins at } t|\text{Coll})$ is the probability that i wins future auction t under collusion, $\Pr(i \text{ wins at } t|\text{NE})$ is the probability that i wins auction t under competition, and $\mathfrak{b}_{i,t}^{\text{Coll}}$ and $\mathfrak{b}_{i,t}^{\text{NE}}$ denote i's bids conditional on the event that i wins under collusion and competition.[46] The expectation in the expression integrates over the realization of costs and bids in the future. Summing the above expression across all auction participants, we obtain an expression that is useful for analyzing the welfare implications of collusion:

$$\begin{aligned}
\sum_i \{V_i^{\text{Coll}} - V_i^D\} &= \mathbb{E}\sum_t [\beta(1-\delta)]^t \{(\mathfrak{b}_{\text{win},t}^{\text{Coll}} - c_{\text{win},t}^{\text{Coll}}) - (\mathfrak{b}_{\text{win},t}^{\text{NE}} - c_{\text{win},t}^{\text{NE}})\} \\
&= \mathbb{E}\sum_t [\beta(1-\delta)]^t \underbrace{(\mathfrak{b}_{\text{win},t}^{\text{Coll}} - \mathfrak{b}_{\text{win},t}^{\text{NE}})}_{\text{price incr.}} - \mathbb{E}\sum_t [\beta(1-\delta)]^t \underbrace{(c_{\text{win},t}^{\text{Coll}} - c_{\text{win},t}^{\text{NE}})}_{\text{cost changes.}}, \qquad (9)
\end{aligned}$$

where $\mathfrak{b}_{\text{win},t}^{\text{Coll}}$ and $c_{\text{win},t}^{\text{Coll}}$ denote the winning bid and the cost of the winner under collusion in future auction t. Similarly, $\mathfrak{b}_{\text{win},t}^{\text{NE}}$ and $c_{\text{win},t}^{\text{NE}}$ denote the winning bid and the cost of the winner under competition. Note that expression (9) has a very intuitive interpretation. The first term on the right-hand side is the discounted sum of expected price elevations under collusion relative to competition. The second term is the discounted sum of expected cost changes (i.e., efficiency changes) under collusion relative to competition. Expression (9) simply states that the sum of differences in continuation values, $\sum_i \{V_i^{\text{Coll}} - V_i^D\}$, should be attributed to either price increases or possible efficiency gains from colluding.

Combining expressions (8) and (9), we can construct an inequality that links the sum of deviation profits to expected price increases:

$$\sum_i [\text{Deviation Profit}] + \mathbb{E}\sum_t [\beta(1-\delta)]^t (c_{\text{win},t}^{\text{Coll}} - c_{\text{win},t}^{\text{NE}}) \leq \mathbb{E}\sum_t [\beta(1-\delta)]^t (\mathfrak{b}_{\text{win},t}^{\text{Coll}} - \mathfrak{b}_{\text{win},t}^{\text{NE}}).$$

[46] Because of the multi-round nature of the auctions, the actual realization of $\mathfrak{b}_{i,t}^{\text{Coll}}$ and $\mathfrak{b}_{i,t}^{\text{NE}}$ may correspond to bidder i's first round bid or later round bids.

If allocation under collusion is equally efficient as under competition, the second term in the left-hand side of the above expression is zero. This would be the case, for example, if allocation under Nash and collusion are both efficient. If allocation under collusion is less efficient, then the second term will be strictly positive. This would be the case if, for example, collusion takes the form of a bid rotation scheme that does not take into account cost differences across colluding firms. In either case, we have

$$\sum_i \text{[Deviation Profit]} \leq \mathbb{E} \sum_t [\beta(1-\delta)]^t (\mathfrak{b}^{\text{Coll}}_{\text{win},t} - \mathfrak{b}^{\text{NE}}_{\text{win},t}). \tag{10}$$

We now use expression (10) to bound damages inflicted by collusion. In particular, we use deviation profits of 856 thousand yen associated with the strategy $x\mathfrak{b}_i^2$ with $x = 97.0\%$ and $\delta = 15\%$ (corresponds to the bottom right column of Table 4) to evaluate the left-hand side of expression (10). Since there are about 7.9 bidders who lose to $i(1)$ by less than 15% in the first round among auctions that proceed to the second round, this gives us a lower bound on total deviation profits of about 6.8 million yen.

The last step in obtaining average price increases under collusion is to specify the discount factor and the rate at which cartels break down. We calibrate β using an annual discount rate of 0.9. Since the average number of auctions in which a firm participates in a given year is about 3.9, we set β to 0.93.[47]. We calibrate δ, the rate of cartel death per year, to 0.17, estimated by Harrington and Wei (2017).[48] We obtain a lower-bound estimate on price increases of about 519 thousand yen, which is about 0.6% of the mean winning bid (1.0% of the median winning bid) of the sample.

Before concluding this subsection, we make a few remarks. First, it is possible to estimate profit gains from using alternative strategies conditional on various observables, such as year, location, project type, etc., without pooling across auctions and bidders with different characteristics. In principle, we can even estimate profit gains firm by firm. However, the estimation of profit gains is quite data intensive, because it requires many auctions that advance to the third round. The number of auctions in our dataset that reach the third round is not enough for us to obtain reliable estimates at a granular level. In the next subsection, we construct an alternative test that is less data intensive, and we apply it to each firm.

Second, a test of competition based on an inequality that pools across observables is, in

[47] $(0.9 \times 0.83)^{1/3.9} \approx 0.93$

[48] There is a small literature that estimates the duration of cartels. The average duration of cartels reported in previous studies is about 5-7 years (See Harrington and Wei, 2017, Levenstein and Suslow (2006)). These duration estimates correspond to a hazard of about 18% to 13% per year assuming a constant hazard.

some ways, stronger than a test based on an inequality that conditions on observables. If the pooled inequality is violated, then the inequality must be violated for some observables.

5.2 Idiosyncratic Factors that Affect Bids

In this section, we provide an alternative test of collusion based on the notion that, in the absence of coordinated bidding, there are idiosyncratic factors that affect how the final bid is determined. In order to motivate our model, we start with a discussion of Dyer and Kagel (1996), a study of bidding behavior based on interviews with executives of the construction industry in the U.S. The study illustrates the complexities of the bid formulation process as well as the seemingly random factors that affect the final bid. For example, in their discussion of the bid formulation process, Dyer and Kagel (1996) note as follows:

> Right up until the moment bids are closed a GC [general contractor] will be working with SCs [subcontractors], confirming the scope of activity associated with the SCs' bids, and accepting/ arguing for cuts in the SCs' bids (a member of the GC's bid team will be stationed at the bid site to fill in bid values moments before the bid closing). It is not uncommon for major SCs' bids to arrive within the last 10-20 minutes before the bid closing, resulting in chaotic last minute interactions with SCs.

They further note that the subcontractors' bids received by the general contractor are often subject to randomness:

> ...failures to bid a GC may result from the chaotic, last minute submission of SCs' bids (and changes in these bids) that characterize the industry, so that an SC may not get through to all the GCs. To the extent this last element dominates, variation in SCs' bids might best be attributed to pure chance.

Dyer and Kagel (1996) also describe a number of other factors that affect bidding, for example, failure to properly include certain aspects of the work in its cost estimates, and being "too greedy", presumably in reference to a bid that is higher than is typical. They report that

> the "mistakes" identified here are, apparently, endemic to the bidding process.

Other studies of construction firms also document evidence of idiosyncratic randomness affecting the outcome of the final bid (See, e.g., Laryea and Hughes, 2008). While it is certainly possible that idiosyncratic randomness affects only a small component of the final bid and that bidders are playing optimally to first-order approximation, the presence of at least some idiosyncratic shocks seems relatively well documented. In order to capture these features of the bidding process, we directly model the bidding strategy of the firms as depending on bidder costs, history of play, and at least one idiosyncratic optimization error. Our model offers a way to incorporate idiosyncratic factors that affect bids in a relatively unrestricted way.

Model of Competitive Bidding with Idiosyncratic Factors We extend the basic model of competitive bidding by adding noise to the bidder's bid. Specifically, we let bidder i' round r bid (normalized by the reserve price) to depend on bidder i's costs, c_i, history of play up to round r, h_i^r as well as a vector of idiosyncratic shocks, $x_i^r \in \mathbb{R}^K$ as follows:

$$b_i^r(c_i, x_i^r; h_i^r). \tag{11}$$

The only difference between this model and the standard model of bidding is the presence of x_i^r. In the following analysis we assume that x_i^r is one-dimensional ($K = 1$), but the analysis remains the same for $K > 1$.

The key substantive assumption that we impose on the null hypothesis of competition (i.e., no coordinated bidding) is that x_i^r is an optimization error that is independent of factors that determine bidder j's bid, b_j^r:

Assumption 1: (Independence) If bidders do not coordinate their bids, x_i^r is independent of factors that determine b_j^r:

$$x_i^r \perp (c_j, x_j^r, h_j^r) \quad (j \neq i),$$

where x_j^r is the idiosyncratic factor that affects bidder j's bid in round r.

This assumption is motivated by the descriptive studies documenting that, in the absence of coordination, the exact bid value is likely to be partly determined by idiosyncratic factors. The term x_i^r captures the (round-r specific) idiosyncratic factor that determines the realization of bidder i's bid and that other bidders do not know about.[49] This is our substantive assumption on competition. We interpret violation of this assumption as suggesting coordinated bidding.

We make a few remarks on this assumption. The first is the relationship between Assumption 1 and Quantal Response Equilibria (QRE) of McKelvey and Palfrey (1995). We show in the Appendix that it is possible to formulate the continuous strategy version of the QRE as expression (11). In particular, because players' strategies place positive density on all possible bids in QRE, we can take x_i^r as the random number that determines the actual realization of the bid. We can then show that x_i^r satisfies Assumption 1. Similarly, if bidders play trembling-hand strategies, x_i^r can be taken as the trembles and Assumption 1 is satisfied.

The second remark is on mixed strategies. While the independence assumption is most natural when bidders are boundedly rational, the assumption can be satisfied with rational bidders if bidders use mixed strategies.[50] Mixed strategies can also be formulated as expression (11) by taking x_i^r as the random number that determines the actual realization of the bid. By construction, x_i^r is independent of b_j^r.

In addition to the independence assumption, we also impose a few technical assumptions. First, we assume that b_i^r is strictly monotone in x_i^r. While this assumption may seem very restrictive, this assumption amounts to a suitable relabeling of the variables in many applications.[51] In particular, we assume that b_i^r is continuously differentiable and the derivative is uniformly bounded. Of course, monotonicity of b_i^r only implies almost everywhere differentiability and not continuous differentiability. However, the stronger assumption of continuous differentiability is needed for constructing our empirical tests which will take the form of regression discontinuity. We also assume that c_i and x_i^r admits density and impose a support condition on c_i and x_i^r.

[49] Correlation in x_i^r across rounds is ruled out by Assumption 1. To see this, note that Assumption 1 implies $x_i^r \perp h_j^r$, where h_j^r is the observed history of bidder j at round r. The history includes the lowest bids from previous rounds. In particular, if bidder i is the lowest bidder in round $r-1$, h_j^r includes b_i^{r-1}. Consequently, $x_i^r \perp h_j^r$ implies $x_i^r \perp b_i^{r-1}$, which also implies $x_i^r \perp x_i^{r-1}$.

[50] See e.g., Hortaçsu and Puller (2008) and Hortaçsu et al. (2019) for evidence of bounded rationality among firms that participate in electricity auctions.

[51] For example, suppose $b_i^r(c_i, x_i^r; h_i^r) = \beta c_i + (x_i^r)^2$. By defining $\tilde{x}_i^r$ as $\tilde{x}_i^r = (x_i^r)^2$, we can express the bidding function as follows: $b_i^r(c_i, \tilde{x}_i^r; h_i^r) = \beta c_i + \tilde{x}_i^r$. Then, $\tilde{x}_i^r$ will still be independent of x_j^r and b_i^r is monotone in $\tilde{x}_i^r$.

Assumption 2 $\frac{\partial b_i^r(c_i, x_i^r, h_i^r)}{\partial x_i^r}$ is continuous with respect to x_i^r and bounded, $m < \frac{\partial b_i^r(c_i, x_i^r, h_i^r)}{\partial x_i^r} < M$ for some $m, M > 0$.

Assumption 3 c_i admits smooth density that is positive and bounded. Moreover, x_i^r admits a conditional density $f_{x_i^r|c_i}$ that is smooth, positive and bounded:

$$0 < m' < f_{c_i}(\cdot), f_{x_i^r|c_i}(\cdot|c_i^r) < M' < \infty$$

for some $m', M' > 0$.

Assumption 4 For each bid b in the support of bidder i's bid distribution and for each c_i, there exists x_i^r such that $b_i^r(x_i^r, c_i; h_i^r) = b$.[52]

Our independence assumption, together with Assumptions 2-4 imply that there exists a completely unpredictable component to the realization of the rival's bids. This implies that bids of two different bidders cannot be linked deterministically, and in particular, it implies that $b_i^r - b_{i(1)}^r$ cannot have a kink in the distribution at $b_i^r - b_{i(1)}^r = 0$ (or at any other point). The following proposition states this formally.

Proposition 1 *Suppose that Assumptions 1 through 4 hold. For each $h > 0$, the probability that $b_i^r - b_{i(1)}^r$ falls in a small band to the left of 0, (i.e., $b_i^r - b_{i(1)}^r \in [-h, 0)$), conditional on the event that $b_i^r - b_{i(1)}^r$ falls within a small band h around 0, (i.e., $b_i^r - b_{i(1)}^r \in [-h, h]$), must converge to 0.5 as h goes to zero:*

$$\lim_{h \to +0} \frac{\Pr\left(b_i^r - b_{i(1)}^r \in [-h, 0)\right)}{\Pr\left(b_i^r - b_{i(1)}^r \in [-h, h]\right)} = 0.5.$$

Moreover,

$$\lim_{h \to +0} \mathbf{E}\left[\mathbf{1}_{\left\{b_i^r - b_{i(1)}^r < 0\right\}} \middle| |b_i^r - b_{i(1)}^r| = h\right] = 0.5. \tag{12}$$

The second expression simply states that the probability of event $b_i^r - b_{i(1)}^r = -h$ conditional on event $|b_i^r - b_{i(1)}^r| = h$, converges to 0.5 as $h \to +0$. The second expression is useful because it is an implication that can be tested directly using a nonparametric regression. Proof of Proposition 1 is in the Appendix.

[52] This assumption does not require the bid to have full support. Bajari and Hortacsu (2005) has a discussion of imposing full support on the realized bids in an auction environment.

For competitive auctions (i.e., auctions that satisfy Assumptions 1 through 4 for all i and r), the conditional expectation of $\mathbf{1}_{\left\{b^r_{i(2)}-b^r_{i(1)}<0\right\}}$ converges to 0.5 as in expression (12). If there are both competitive and collusive auctions in the data, the conditional expectation of $\mathbf{1}_{\left\{b^r_{i(2)}-b^r_{i(1)}<0\right\}}$ will be a weighted average of two conditional expectations: one for the set of competitive auctions and the other for the set of collusive auctions with weights given by the share of competitive and collusive auctions. Given that the conditional expectation must be 0.5 for the set of competitive auctions and given that it must be nonnegative for collusive auctions, the extent to which the conditional expectation is different from 0.5 gives us a bound on the share of competitive auctions. The following corollary states this formally.[53]

Corollary *Let $s^r(h)$ ($r = 2, 3$) denote the share of competitive auctions among the set of auctions for which $b^r_{i(2)} - b^r_{i(1)} = h$. Let $s^r(0)$ be the* $\limsup$ *of $s^r(h)$ as $h \to 0$. Then,*

$$s^r(0) \leq 2 \times \limsup_{h \to +0} \mathbf{E}\left[\mathbf{1}_{\left\{b^r_{i(2)}-b^r_{i(1)}<0\right\}} \middle| |b^r_{i(2)} - b^r_{i(1)}| = h\right].$$

Note that when the limit of the conditional expectation is 0.5, the bound on the share of competitive auctions is 1. The bound is informative when the limit of the conditional expectation is less than 0.5.

Finally, another implication of our assumptions is that the set of bidders who are marginally outbid by $i(1)$ in the reauction should, on average, look similar to the set of bidders who marginally outbid $i(1)$. In particular, the average winning bid of the five preceding auctions we defined toward the end of Section 4, b^{before}_i, should be the same, in expectation, for the former and the latter set of bidders.[54] The next proposition states this claim formally.

Proposition 2 *Suppose Assumption 1 through Assumption 4 hold. Assume also that b^{before}_i has finite moments and that $(b^{before}_i, b^2_i - b^2_{i(1)})$ has continuous and positive joint density.*

[53] The following bound is computed under the extreme assumption that the conditional expectation is 0 for collusive auctions (which corresponds to the highest share of competitive auctions). This assumption is reasonable if collusion is always all-inclusive. However, the conditional expectation for collusive auctions would be significantly higher than 0 if collusion is partial. In this sense, the bound is also informative about the prevalence of partial cartels.

[54] Recall that, for each auction n, we define $b^{before}_{i,n}$ as follows:

$$b^{before}_{i,n} = \frac{1}{5}\sum_{m=n-5}^{m=n-1} b^{win}_m,$$

where b^{win}_m ($m \in \{n-5 \cdots n-1\}$) are the winning bids of the five auctions preceding auction n.

Then, the average of b_i^{before} should be the same for the set of bidders who marginally outbid $i(1)$ in the second round and the set of bidders who are marginally outbid by $i(1)$ in the second round:

$$\lim_{b_i^2 - b_{i(1)}^2 \to +0} \mathbf{E}\left[b_i^{before} \middle| b_i^2 - b_{i(1)}^2 \right] = \lim_{b_i^2 - b_{i(1)}^2 \to -0} \mathbf{E}\left[b_i^{before} \middle| b_i^2 - b_{i(1)}^2 \right].$$

This proposition justifies our regression discontinuity analysis for b_i^{before} in Section 4.[55]

5.2.1 Formal Test

We now use Proposition 1 to construct a formal test of the null. In order to construct our test, note that expression (12) can be expressed as $\lim_{X \to +0} \mathbf{E}[Y|X] = 0.5$, where $Y = \mathbf{1}_{\{b_i^r - b_{i(1)}^r < 0\}}$, $X = |b_i^r - b_{i(1)}^r|$, and $\mathbf{1}_E$ is an indicator function that is equal to one if and only if event E is true. Note that b_i^r and $b_{i(1)}^r$ are normalized bids. Consistent estimates of a conditional expectation of the form $\mathbf{E}[Y|X]$ at $X = +0$ can be obtained by a local polynomial regression. In particular, consider the minimizer of the following objective function

$$\widehat{\beta} = \underset{(b_0 \cdots b_p) \in \mathbb{R}^{p+1}}{\arg\min} \sum_{n=1}^{N} (Y_n - b_0 - b_1 X_n - \cdots - b_p X_n^p)^2 K\left(\frac{X_n}{h_N}\right), \tag{13}$$

where $K(\cdot)$ is a kernel, h_N is a bandwidth, p is the order of the local polynomial regression function. The first element of $\widehat{\beta}$ is a consistent estimate of $\lim_{X \to +0} \mathbf{E}[Y|X]$ if $h_N N^{1/(2p+3)} \to 0$ (Fan and Gijbels, 1992). In practice, we use $p = 1$ and follow the bias correction method proposed in Calonico et al. (2018, 2019) to compute the confidence interval. We use the Epanechnikov kernel for $K(\cdot)$ and a mean square error optimal bandwidth for h_N. This allows us to test expression (12).

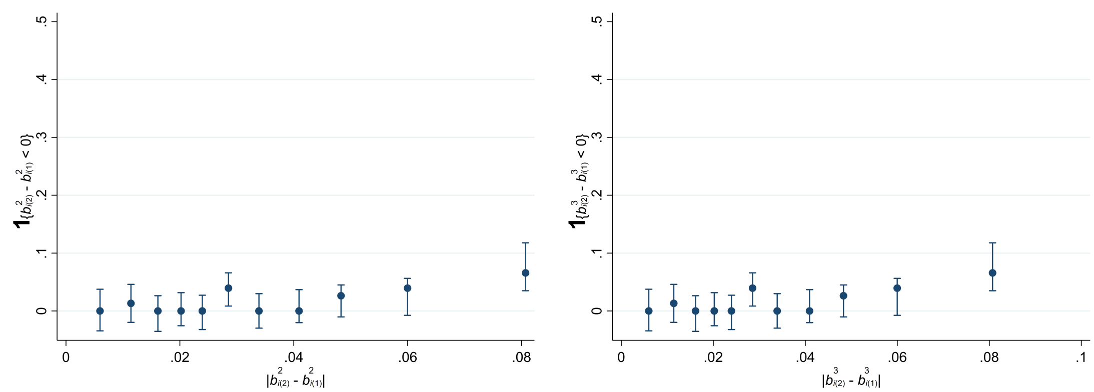
Results for All Samples We first test for expression (12) with $r = 2$ using all auctions in which there is a second round of bidding. For each auction n, we compute $Y_n = \mathbf{1}_{\{b_{i(2)}^2 - b_{i(1)}^2 < 0\}}$ and $X_n = |b_{i(2)}^2 - b_{i(1)}^2|$ and obtain an estimate of $\lim_{X \to +0} \mathbf{E}[Y|\ X]$ using expression (13). Column (1) of Table 5 presents our results. We find that the point estimate is 0.014 and the standard error is 0.002. We find that 0.5 is not contained in the

[55] An analogous result for b_i^{after} will not hold if winning an auction today affects how the bidder bids in the future.

	(1)	(2)
	Round 2	Round 3
Estimate	0.0136	-0.0063
	(0.0024)	(0.0061)
Bandwidth	0.1169	0.0143
Sample Size	7,839	902

Note: The estimate is computed using a local linear regression with bias correction (Calonico et al., 2018, 2019). We use the Epanechnikov kernel. Standard errors are reported in parenthesis.

Table 5: Estimate of $\mathbf{1}_{\left\{b^r_{i(2)} - b^r_{i(1)} < 0\right\}}$ conditional on $|b^r_{i(2)} - b^r_{i(1)}| \to 0$.



The horizontal axis is $|b^2_{i(2)} - b^2_{i(1)}|$ (Left Panel) and $|b^3_{i(2)} - b^3_{i(1)}|$ (Right Panel). The bars in the graphs indicate 95% confidence intervals.

Figure 5: Binned Scatter Plots of $\mathbf{1}_{\{b^2_{i(2)} - b^2_{i(1)} < 0\}}$ (Left Panel) and $\mathbf{1}_{\{b^3_{i(2)} - b^3_{i(1)} < 0\}}$ (Right Panel).

95% confidence interval which leads us to reject the null. The point estimate suggests a (local) upper bound on competitive auctions of 2.8% (See Corollary). The result suggests that almost all auctions that reach the second round are uncompetitive, and moreover, almost all of the bidding rings are all-inclusive.[56] The left panel of Figure 5 is the corresponding binned scatter plot of Y_n. The figure shows that the bin averages are far from 0.5.

Next, we test for expression (12) with $r = 3$ using the set of auctions in which there

[56]If the bidding ring is partial, we would expect competitive bidders to outbid $i(1)$ some of the time, resulting in the conditional expectation to converge to a number that is higher than what we estimate.

is a third round of bidding. For each of these auctions, we set $Y_n = \mathbf{1}_{\{b^3_{i(2)} - b^3_{i(1)} < 0\}}$ and $X_n = |b^3_{i(2)} - b^3_{i(1)}|$ and estimate the conditional expectation, $\mathbf{E}[Y|X]$, at $X = 0$ using expression (13). The results are reported in Column (2) of Table 5. We find that the point estimate is -0.006 and the standard error is 0.006. Again, we reject the null that $\mathbf{E}[Y|X]$ converges to 0.5 as $X \to +0$. The right panel of Figure 5 is the corresponding binned scatter plot, which again shows that the bin averages are far from 0.5.

Results for Each Firm We now apply our test firm by firm. Specifically, for each firm i, we collect all auctions in which 1) firm i participates; 2) the auction proceeds to the second round; 3) i is not $i(1)$ in the first round of the auction, and 4) i bids in the second round. We then compute $Y_n = \mathbf{1}_{\{b^2_i - b^2_{i(1)} < 0\}}$ and $X_n = |b^2_i - b^2_{i(1)}|$ for each auction. If there are more than 10 auctions that satisfy the four conditions above, we test for expression (12) for bidder i. We are able to run our test on a total of 1,098 firms.

Table 6 reports our results. We report the results by groups of 50 firms, starting from the set of firms that win the most number of auctions during the sample period. In other words, the first row reports the results for the 50 largest firms by the number of auctions won, the second row reports the results for the next 50 largest firms, and so on.

Column (1) of the table reports the number of firms for which we compute the test statistic and Column (2) reports the number of firms for which we reject expression (12). We find that a total of 1,066 firms bid in a manner inconsistent with our benchmark of competitive bidding (out of 1,096 firms for whom we compute the test statistic). In particular, we find that 613 firms are uncompetitive out of the top 1,000 firms. While these results do not account for the fact that we are testing multiple hypotheses, controlling for the false discovery rate at 5% only reduces the number of uncompetitive firms to 1,059.[57]

Column (3) of the table reports the rejection probability, computed by dividing the numbers in Column (2) by 50. The rejection probability is around 75% for the largest 1-400 firms and it is around 50% for the 401-750 largest firms. The number remains above 20% for all groups larger than group 1,450-1,500. If we compute the rejection probability taking the number of firms tested as the denominator, it is over 95%.

Columns (4)-(6) of Table 6 report the average number of times a firm participates (Column (4)), the average number of times a firm bids in Round 2 (Column (5)), and the average total award amount (Column (6)). These averages are taken over all of the firms within a

[57]We use the procedure proposed by Benjamini and Yekutieli (2001) to control for the false discovery rate. See Schurter (2017) for an approach that controls for the asymptotic family-wise error rate in a very similar setting as ours.

group. Column (7) reports the average number of auctions that satisfy criteria 1) - 4) for computing the test statistic (for the set of firms that we compute the test statistic). For Column (7), the averages are taken over a smaller number of firms.

The total number of auctions awarded to uncompetitive firms that we identify is 15,583, or close to 37.1% of the total number of auctions in our sample. The total award amount of these auctions is about \$18.6 billion, or 44.2 % of the total value of the auctions in the sample.

While our dataset accounts only for public construction projects procured at the national level, firms that we identify as uncompetitive are also active in local public procurement auctions. Given that the total award amount of public construction projects in Japan is about \$200 billion per year, or approximately 4% of Japan's GDP, cartel activity among construction firms may have economy-wide significance.[58]

Interaction among the Uncompetitive Firms In order to provide information on the participation patterns of the firms that we identify as uncompetitive, we compute summary statistics that we report in Table 3 of Section 3, but restricting the sample to only those that we identify as uncompetitive. Letting I denote the set of 1,066 firms that we reject the null of competition, we compute the average number of other bidders in I that a firm bids with, the number of unique bidders in I that a bidder bids with, and the shares of close competitors.

Table 7 reports the results separately for four sets of firms grouped by the total number of auctions in which the firm participates in our sample. The top row corresponds to the results for the 30 largest firms in I that participate in more than 500 auctions. The second row corresponds to the set of 537 firms that bid on 100 - 500 auctions, and so on.

Column (1) reports the average number of auctions in which a firm participates. Column (2) reports the average number of other uncompetitive bidders that bidders face. We find that the average number of uncompetitive rival bidders is above 6 for the largest group and it is around 4-5 for all other groups. Given that the average total number of competitors that a bidder faces (competitive or not) in an auction is around 9 (see Table 3), this result implies that about half of the rival bidders that an uncompetitive bidder faces are also in I.

Column (3) reports the number of unique opponents in I against whom a firm bids in the sample. In Columns (4) - (8), we report the share of auctions in which the bidder's

[58]Also, the rules governing procurement auctions for municipalities and prefectures are very similar to the ones used by the Ministry of Land, Infrastructure, and Transportation.

Firms	(1) # Firms Tested	(2) # Firms Reject	(3) (2) / # Firms	(4) Particip. Round 1	(5) Particip. Round 2	(6) Award Amt.	(7) Obs.
1 - 50	46	43	86%	908.8	85.2	8,600	59.0
51 - 100	46	42	84%	447.2	61.8	4,660	42.5
101 - 150	44	41	82%	315.0	43.8	2,908	30.9
151 - 200	39	37	74%	244.8	31.0	2,331	23.8
201 - 250	39	38	76%	206.8	27.4	1,883	21.5
251 - 300	40	37	74%	204.3	29.0	2,318	22.4
301 - 350	39	38	76%	193.0	27.8	1,729	21.4
351 - 400	36	35	70%	162.9	22.1	1,604	18.3
401 - 450	35	32	64%	176.0	21.1	1,915	17.7
451 - 500	31	31	62%	150.3	21.5	1,634	20.3
501 - 550	24	20	40%	126.7	17.7	1,204	16.6
551 - 600	28	27	54%	127.1	18.0	1,232	16.6
601 - 650	29	29	58%	130.2	17.5	1,109	16.0
651 - 700	24	23	46%	117.7	16.9	1,081	17.3
701 - 750	24	23	46%	111.4	15.9	1,291	15.8
751 - 800	23	23	46%	114.5	15.4	1,096	17.5
801 - 850	24	23	46%	111.8	16.8	1,290	16.2
851 - 900	21	21	42%	95.8	14.2	1,293	14.6
901 - 950	27	27	54%	107.7	16.4	756	15.5
951 - 1000	23	23	46%	108.5	14.3	945	15.0
1001 - 1050	18	17	34%	76.6	12.0	589	14.9
1051 - 1100	20	19	38%	91.6	14.3	726	16.1
1101 - 1150	11	11	22%	79.4	11.5	602	15.3
1151 - 1200	18	18	36%	83.8	13.2	568	13.9
1201 - 1250	13	13	26%	73.4	10.8	602	16.5
1251 - 1300	16	16	32%	79.3	12.7	675	14.1
1301 - 1350	18	18	36%	72.7	11.2	563	13.1
1351 - 1400	9	9	18%	71.2	9.8	511	17.2
1401 - 1450	16	16	32%	72.0	12.1	654	15.4
1451 - 1500	9	9	18%	69.5	9.3	513	14.1
1501 - 1550	10	10	20%	64.7	9.6	521	14.8
1551 - 1600	15	15	30%	64.6	9.7	401	12.7
1601 - 1650	10	10	20%	57.3	9.3	431	12.6
1651 - 1700	8	8	16%	58.4	9.0	427	15.5
1701 - 1750	10	10	20%	63.2	10.2	482	13.7
1751 - 1800	13	13	26%	67.3	10.5	626	14.5
1801 - 1850	13	13	26%	65.7	9.7	519	13.2
1851 - 1900	5	5	10%	48.7	6.9	310	13.8
1901 - 1950	8	8	16%	58.4	9.3	327	13.6
1951 - 21287	216	215	1%	10.3	1.7	66	12.5
Total	1098	1066	5%	21.2	3.2	170	19.7

Table 6: Test of Competition for Each Firm by Groups of 50 Firms.

Group	(1) Partici-pation	(2) Rival Bidders	(3) Unique Rivals	(4) S_1	(5) S_2	(6) S_4	(7) S_{10}	(8) S_{15}	(9) Obs.
500+	825.7	6.1	151.8	59.0	53.1	42.8	23.9	10.7	30
	(310.7)	(1.0)	(88.9)	(16.3)	(15.9)	(13.5)	(8.3)	(5.2)	
100 - 500	183.7	5.1	103.5	37.7	33.0	27.1	16.4	11.3	537
	(82.5)	(1.7)	(62.5)	(19.2)	(18.2)	(16.3)	(12.6)	(10.9)	
50 - 100	74.3	4.7	51.1	46.8	40.1	31.8	16.9	10.3	387
	(13.8)	(1.7)	(28.8)	(19.4)	(17.7)	(15.6)	(9.5)	(7.9)	
15 - 50	40.8	4.3	31.6	52.0	43.7	34.0	17.6	12.5	112
	(7.5)	(1.8)	(17.6)	(19.7)	(18.3)	(15.9)	(13.6)	(16.0)	
Total	147.1	4.9	78.3	43.1	37.3	30.0	16.9	11.0	1,066
	(151.3)	(1.7)	(59.0)	(20.1)	(18.6)	(16.3)	(11.6)	(10.5)	

Table 7: Summary Statistics of Bidders Identified as Non-Competitive.

close opponents participate. Close opponents are defined in the same way as in Table 3, but restricted to other bidders in I. Column (4) corresponds to the share of the most frequent rival among I and Column (5) corresponds to the share of the second most frequent rival in I. Column (6), (7), and (8) correspond to the share of the fourth, tenth and the fifteenth most frequent rival, respectively. We find that the share of the closest rival (S_1) is around 40% on average, which means that a bidder bids with its closest rival in I on about 40% of the auctions. The share for the fifteenth closest rival (S_{15}) is about 10%, implying that the fifteenth closest rival participates in about 10% of the auctions. The table illustrates the repeated nature of the interaction among the set of bidders that we identify as uncompetitive.

Discussion of Our Detection Method We have discussed several ways of detecting collusion in this section. We now briefly discuss whether our ideas are useful even when bidders become aware of our detection strategy.

As a general point, proposing a detection method can be thought of as putting an additional constraint on the pattern of bidding that a ring can safely engage in. Even if bidding rings respond to a new detection method, the method can still serve a useful purpose by making it potentially harder to sustain collusion, lessening the damages from collusion, or making it easier to detect collusion with existing methods. For example, one simple way for rings to avoid being detected by our method is to decrease their bids so that the auction ends in the first round. This will diminish the damages from collusion (as the bids have to be lowered, on average) and decrease the incentive to collude.

There are other ways to avoid our detection method, such as changing the identity of the lowest bidder across rounds or having the lowest bidder bid substantially less than everybody else. These responses are likely to impose substantial costs on the bidding ring or make detection easier by other means. For example, if the bidding ring changed the identity of the lowest bidder from round to round, this would require at least two bidders to incur the cost of estimating the project. It would also incentivize the designated bidder in Round 1 to bid more aggressively and win the auction in Round 1 as a different firm will win the project in Round 2. Hence, designating multiple winners may come at a cost, potentially making it harder to sustain collusion. Alternatively, if one bidder submits a bid that is substantially less than everybody else's, this would be quite suspicious if the bidding ring is all-inclusive. It may put the ring at risk of being detected by other methods.

6 Conclusion

In this paper, we document large-scale collusion among construction firms in Japan using bidding data in reauctions. Although the exact details of the auction vary from setting to setting, rebidding is a common feature in many government procurement projects. Our approach may provide a useful starting point when screening for collusion in other settings.

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Appendix

Omitted Proofs

Proof of the Lemma

Proof. We first prove that the distribution of bidder i's cost, c_i, conditional on $\{\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, \{r < \min_j \mathfrak{b}_j^1\}\}$ is independent of whether or not the auction ends in the second round. Let $g(\cdot)$ denote the density of c_i and $F_r(\cdot)$ denote the distribution function of the reserve price. Then,

$$
\begin{aligned}
& g(c_i | \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, \{r < \min_j \mathfrak{b}_j^1\}, \{r < \min_j \mathfrak{b}_j^2\}) \\
&= \frac{\Pr(c_i, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, \{r < \min_j \mathfrak{b}_j^1\}, \{r < \min_j \mathfrak{b}_j^2\})}{\Pr(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, \{r < \min_j \mathfrak{b}_j^1\}, \{r < \min_j \mathfrak{b}_j^2\})} \\
&= \frac{\Pr(r < \min\{\min_j \mathfrak{b}_j^1, \min_j \mathfrak{b}_j^2\} | c_i, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) \times \Pr(c_i, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2)}{\Pr(r < \min\{\min_j \mathfrak{b}_j^1, \min_j \mathfrak{b}_j^2\} | \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) \times \Pr(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2)}.
\end{aligned}
$$

Given that r is independent of c_i, it is also independent of $\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2$, and $\min_j \mathfrak{b}_j^2$. Hence, the last expression is simplified as follows:

$$
\begin{aligned}
& \frac{\Pr(c_i, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) \times \overbrace{F_r(\min\{\min_j \mathfrak{b}_j^1, \min_j \mathfrak{b}_j^2\})}^{\text{Pr. go to Round 3}}}{\Pr(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) \times F_r(\min\{\min_j \mathfrak{b}_j^1, \min_j \mathfrak{b}_j^2\})} \\
&= \frac{\Pr(c_i, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2)}{\Pr(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2)} \left(= g(c_i | \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2)\right) \\
&= \frac{\Pr(c_i, \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) \times \overbrace{\max\{0, F_r(\min_j \mathfrak{b}_j^1) - F_r(\min_j \mathfrak{b}_j^2)\}}^{\text{Pr. end in Round 2}}}{\Pr(\mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2) \times \max\{0, F_r(\min_j \mathfrak{b}_j^1) - F_r(\min_j \mathfrak{b}_j^2)\}} \\
&= g(c_i | \mathfrak{b}_i^1, \min_j \mathfrak{b}_j^1, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, \{\min_j \mathfrak{b}_j^2 \leqq r < \min_j \mathfrak{b}_j^1\}).
\end{aligned}
$$

Hence, the first part of the lemma follows.

We now show the second part. By using an argument similar to above, we have

$$\begin{aligned}
&\mathbf{E}_{\mathcal{H}}[c_i | \{r \geqq \min_j \mathfrak{b}_j^2\}, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, (\tilde{W}^2 - W^2)], \\
=&\mathbf{E}_{\mathcal{H}}[c_i | \{r < \min_j \mathfrak{b}_j^2\}, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2].
\end{aligned}$$

Using the restriction that bidders do not bid strictly below their costs, we obtain

$$\begin{aligned}
&\mathbf{E}_{\mathcal{H}}[c_i | \{r \geqq \min_j \mathfrak{b}_j^2\}, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2, (\tilde{W}^2 - W^2)], \\
\leqq&\mathbf{E}_{\mathcal{H}}[\mathfrak{b}_i^3 | \{r < \min_j \mathfrak{b}_j^2\}, \mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2], \\
=&h_{\mathcal{H}}(\mathfrak{b}_i^2, \min_j \mathfrak{b}_j^2).
\end{aligned}$$

Integrating both sides over $\mathfrak{b}_i^2$ and $\min_j \mathfrak{b}_j^2$ for the event such that $(\tilde{W}^2 - W^2) \cap \{r \geqq \min_j \mathfrak{b}_j^2\}, \mathcal{H}$, we have the second part of the lemma. ■

Proof of Proposition 1

Proof. Let $f_{b_i^r - b_{i(1)}^r}$ denote the density of $b_i^r - b_{i(1)}^r$. Assumptions 1 through 4 imply that $f_{b_i^r - b_{i(1)}^r}$ exists, and that it is continuous, strictly positive, and bounded above by some constant. This is shown in Online Appendix V. In what follows, we take this fact as given.

Take any $\varepsilon > 0$. Define ϵ as follows:

$$\epsilon = \varepsilon \times f_{b_i^r - b_{i(1)}^r}(0)(> 0).$$

Now take $\delta_\varepsilon > 0$ so that the following is satisfied for all $|t| < \delta_\varepsilon$:

$$|f_{b_i^r - b_{i(1)}^r}(t) - f_{b_i^r - b_{i(1)}^r}(0)| < \epsilon.$$

This is possible because $f_{b_i^r - b_{i(1)}^r}$ is continous. For each $\delta < \delta_\varepsilon$, consider the probability that $b_i^r - b_{i(1)}^r$ falls in the interval $[-\delta, 0)$:

$$\begin{aligned}
\Pr\left(b_i^r - b_{i(1)}^r \in [-\delta, 0)\right) &= \delta \times f_{b_i^r - b_{i(1)}^r}(0) + \int_{t=-\delta}^{t=0} \left(f_{b_i^r - b_{i(1)}^r}(t) - f_{b_i^r - b_{i(1)}^r}(0)\right) dt. \\
&\leq \delta \times f_{b_i^r - b_{i(1)}^r}(0) + \int_{t=-\delta}^{t=0} \epsilon dt \\
&= \delta \times f_{b_i^r - b_{i(1)}^r}(0)(1 + \varepsilon),
\end{aligned}$$

Similarly, consider the probability that $b_i^r - b_{i(1)}^r$ falls in an interval $[-\delta, \delta)$:

$$\begin{aligned}\Pr\left(b_i^r - b_{i(1)}^r \in [-\delta, \delta)\right) &= 2\delta \times f_{b_i^r - b_{i(1)}^r}(0) + \int_{t=-\delta}^{t=\delta} \left(f_{b_i^r - b_{i(1)}^r}(t) - f_{b_i^r - b_{i(1)}^r}(0)\right) dt. \\ &\geq 2\delta \times f_{b_i^r - b_{i(1)}^r}(0)(1-\varepsilon),\end{aligned}$$

Hence,

$$\frac{\Pr\left(b_i^r - b_{i(1)}^r \in [-\delta, 0)\right)}{\Pr\left(b_i^r - b_{i(1)}^r \in [-\delta, \delta]\right)} \geq \frac{1+\varepsilon}{2(1-\varepsilon)},$$

where the right hand side converges to $1/2$ as ε converges to 0. Hence, we have shown that

$$\liminf_{h \to 0} \frac{\Pr\left(b_i^r - b_{i(1)}^r \in [-h, 0)\right)}{\Pr\left(b_i^r - b_{i(1)}^r \in [-h, h]\right)} \geq 1/2.$$

We can similarly bound the $\limsup$ by $1/2$ which concludes the proof.

We now show the second part of the proposition:

$$\lim_{h \to 0} \mathbf{E}\left[\mathbf{1}_{\left\{b_i^r < b_{i(1)}^r\right\}} \,|\, \left\|b_i^r - b_{i(1)}^r\right\| = h\right] = \frac{1}{2}.$$

Note that

$$\mathbf{E}\left[\mathbf{1}_{\left\{b_i^r < b_{i(1)}^r\right\}} \,|\, \left\|b_i^r - b_{i(1)}^r\right\| = h\right] = \frac{f_{b_i^r - b_{i(1)}^r}(-h)}{f_{b_i^r - b_{i(1)}^r}(h) + f_{b_i^r - b_{i(1)}^r}(-h)}.$$

For any ε define ϵ and δ_ϵ as before. Then, for each $\delta < \delta_\varepsilon$,

$$\begin{aligned} f_{b_i^r - b_{i(1)}^r}(0)(1-\varepsilon) \leq \quad & f_{b_i^r - b_{i(1)}^r}(-\delta), \\ & f_{b_i^r - b_{i(1)}^r}(\delta) \quad \leq f_{b_i^r - b_{i(1)}^r}(0)(1+\varepsilon)\end{aligned}$$

Hence,

$$\frac{1-\varepsilon}{2(1+\varepsilon)} \leq \frac{f_{b_i^r - b_{i(1)}^r}(-h)}{f_{b_i^r - b_{i(1)}^r}(h) + f_{b_i^r - b_{i(1)}^r}(-h)} \leq \frac{1+\varepsilon}{2(1-\varepsilon)},$$

where both sides converges to $1/2$ as ε converges to 0. This concludes the proof. ■

Proof of the Continuous Strategy Version of the Quantal Response Equilibria

Proof. We show that the continuous strategy version of the Quantal Response Equilibria of McKelvey and Palfrey (1995) satisfy the independence assumption.

The continuous action version of the Quantal Response Equilibria requires, for each bidder i, to bid according to density $g_i^r(\cdot)$ given by

$$g_i^r(b_i^r) = \frac{e^{\lambda \pi_i^r(b_i^r)}}{\int_0^\infty e^{\lambda \pi_i^r(b')} db'},$$

where $\pi_i(b_i^r)$ denotes i's expected payoff from bidding b_i^r in Round r. The expected payoff includes both the payoff from winning in round r as well as any continuation value from future rounds. The expected profit, $\pi_i^r(\cdot)$, depends on the bidding strategy of others, so it is an equilibrium object that depends on bidder i's costs, c_i, and the lowest bids from previous rounds, $\{\min_j \mathfrak{b}_j^p\}_{p=1}^{r-1}$. Let u be a random variable that is distributed uniform $[0, 1)$. We can express the bidding function as

$$b_i^r = (G_i^r)^{-1}(u; c_i, \{\min_j \mathfrak{b}_j^1\}_{p=1}^{r-1}),$$

where $(G_i^r)^{-1}(\cdot)$ is the inverse function of $G_i^r(\cdot)$ defined as follows:

$$G_i^r(y) = \frac{\int_0^y e^{\lambda \pi_i^r(b')} db'}{\int_0^\infty e^{\lambda \pi_i^r(b')} db'}.$$

The intuition is that bidders bid all possible bids with positive density and u is simply the random number that determines the actual bid that is played. Given that G_i^r is strictly increasing, continuous and its image is the interval $[0, 1)$, its inverse is well-defined on $[0, 1)$.

Now take the independence factor as the uniform random variable u. By construction, it is independent of all random variables in the model, including those that determine bidder j's bid. u is also independent across rounds. ■

Proof of Proposition 2

Proof. Let $\{h_k\}$ be any positive sequence converging to 0. For any $\varepsilon > 0$, define $U_k^\varepsilon \subset \mathbb{R}^+$ and $V_k^\varepsilon \subset \mathbb{R}^+$ as follows:

$$U_k^\varepsilon = \left\{ x \in \mathbb{R}^+ \left| \| \frac{1}{h_k} \Pr\left(b_j^2 - b_{i(1)}^2 \in (0, h_k] | b_{j,n}^{before} = x \right) - f_{b_j^2 - b_{i(1)}^2 | b_{j,n}^{before}}(0|x) \| < \varepsilon \right. \right\}$$

$$V_k^\varepsilon = \left\{ x \in \mathbb{R}^+ \left| \| \frac{1}{h_k} \Pr\left(b_j^2 - b_{i(1)}^2 \in [-h_k, 0) | b_{j,n}^{before} = x \right) - f_{b_j^2 - b_{i(1)}^2 | b_{j,n}^{before}}(0|x) \| < \varepsilon \right. \right\}$$

Note that $U_k^\varepsilon \nearrow \mathbb{R}^+$ and $V_k^\varepsilon \nearrow \mathbb{R}^+$ (as $k \to +\infty$) because $(b_i^{before}, b_i^2 - b_{i(1)}^2)$ has continuous density. Now,

$$\begin{aligned}
&\mathbf{E}\left[b_{j,n}^{before} \middle| b_j^2 - b_{i(1)}^2 \in [-h_k, 0) \right] \\
&\quad = \int x f_{b_{j,n}^{before}}(x | b_j^2 - b_{i(1)}^2 \in [-h_k, 0)) dx \\
&\quad = \int_{U_k^\varepsilon \cap V_k^\varepsilon} x f_{b_{j,n}^{before}}(x | b_j^2 - b_{i(1)}^2 \in [-h_k, 0)) dx \\
&\qquad + \int_{\mathbb{R} - U_k^\varepsilon \cap V_k^\varepsilon} x f_{b_{j,n}^{before}}(x | b_j^2 - b_{i(1)}^2 \in [-h_k, 0)) dx.
\end{aligned}$$

As $b_{j,n}^{before}$ has finite moments, the second term can be made less than ε for k large, by dominated convergence. Now consider the first term;

$$\begin{aligned}
&\int_{U_k^\varepsilon \cap V_k^\varepsilon} x f_{b_{j,n}^{before}}(x | b_j^2 - b_{i(1)}^2 \in [-h_k, 0)) dx \\
= &\int_{U_k^\varepsilon \cap V_k^\varepsilon} x \frac{(1/h_k) \Pr(b_j^2 - b_{i(1)}^2 \in [-h_k, 0) | b_{j,n}^{before} = x)}{(1/h_k) \Pr(b_j^2 - b_{i(1)}^2 \in [-h_k, 0))} f_{b_{j,n}^{before}}(x) dx \\
= &\int_{U_k^\varepsilon \cap V_k^\varepsilon} x \frac{f_{b_j^2 - b_{i(1)}^2 | b_{j,n}^{before}}(0|x)}{f_{b_j^2 - b_{i(1)}^2}(0)} f_{b_{j,n}^{before}}(x) dx + O(\varepsilon),
\end{aligned}$$

as $k \to \infty$. The last expression holds because $(b_i^{before}, b_i^2 - b_{i(1)}^2)$ has positive density.

Similarly,

$$\begin{aligned}
&\mathbf{E}\left[b_{j,n}^{before} \middle| b_j^2 - b_{i(1)}^2 \in (0, h_k] \right] \\
&\qquad = \int_{U_k^\varepsilon \cap V_k^\varepsilon} x \frac{f_{b_j^2 - b_{i(1)}^2 | b_{j,n}^{before}}(0|x)}{f_{b_j^2 - b_{i(1)}^2}(0)} f_{b_{j,n}^{before}}(x) dx + O(\varepsilon),
\end{aligned}$$

Hence, $\mathbf{E}\left[b_{j,n}^{before} \middle| b_j^2 - b_{i(1)}^2 \in [-h_k, 0) \right]$ and $\mathbf{E}\left[b_{j,n}^{before} \middle| b_j^2 - b_{i(1)}^2 \in (0, h_k] \right]$ can be made as small as desired as $k \to \infty$. ■

For Online Publication

Online Appendix I Sample Statistics on Attrition and Win Rates by Rank and Round

The second and third rounds of bidding are open only to firms who submit valid first-round bids. However, this fact does not mean that bidders who bid in the first round must bid in the second and third rounds. In fact, a non-negligible proportion of bidders decide to drop out of the auction. Table OA.1 reports the summary statistics on the number of valid bids in the reauctions by the rank of the bidder in the initial round. The first row of the table reports the number of bids in Round 2 that we would observe in the data if there were no attrition between the initial and second rounds. The second row reports the number of actual bids in Round 2 and the third row reports the corresponding attrition rate. The rate of attrition is about 6.8% on average, although it is much lower for $i(1)$, at 0.4%. Rows 4 through 6 report the corresponding statistics for the third round. We find that the attrition rate is much higher, at 26.4%.

Table OA.2 reports summary statistics on the winning probability by rank and by terminal round. The first row corresponds to the number of second-round bids that we would observe (among auctions that end in Round 2) if there were no attrition. The numbers in

	Round	$i(1)$	$i(2)$	$i(3)$	$i(4)$	$i(5)$	All
N		8,387	8,363	8,242	8,091	7,916	78,812
Valid Bid	2	8,351	7,888	7,684	7,463	7,288	73,488
Attrition Rate		0.4%	5.7%	6.8%	7.8%	7.9%	6.8%
N		1,249	1,244	1,228	1,207	1,181	11,305
Valid Bid	3	1,230	918	876	854	814	8,319
Attrition Rate		1.5%	26.2%	28.7%	29.2%	31.1%	26.4%

The table reports how many bidders submit bids in Round 2 and Round 3 conditional on submitting a bid in Round 1. The first column corresponds to bidders who are $i(1)$. Out of 8,387 cases that go to the second round, 8,351 $i(1)$ bidders submit a valid bid in Round 2. The table also reports that, out of 1,249 cases that go to the third round, 1,230 $i(1)$ bidders submit a valid bid in Round 3. The attrition rate is very low for $i(1)$ and it is significantly higher for all other positions. The sample size for $i(1)$ is larger than that for $i(2)$ because some auctions only receive one valid bid.

Table OA.1: Attrition by Rank of Bidder in Round 1.

	Round	$i(1)$	$i(2)$	$i(3)$	$i(4)$	$i(5)$	$i(6)+$
N		7,138	7,119	7,014	6,884	6,735	35,697
Won	2	6,895	124	41	22	17	39
Win Rate		96.6%	1.7%	0.6%	0.3%	0.2%	0.1%
N		1,249	1,244	1,228	1,207	1,181	5,649
Won	3	1,191	23	9	7	3	16
Win Rate		95.4%	1.8%	0.7%	0.6%	0.3%	0.3%

The first three rows correspond to auctions that end in Round 2. The last three rows correspond to auctions that reach Round 3. We report the number of bids that we would observe in the absence of attrition (row 1 and row 4), the number of winning bids (row 2 and row 5) and the win rate (row 3 and row 6).

Table OA.2: Win Rate by Round and Rank.

this row are smaller than the numbers in the first row of Table OA.1 because the first row of Table OA.2 focuses on auctions that end in the second round (as opposed to reach the second round). The second row of Table OA.2 corresponds to the number of winning bids. The third row is the win rate. We find that, among the set of auctions that end in Round 2, the win rate of $i(1)$ is 96.6%. Rows 4-6 correspond to the statistics for auctions that reach Round 3. Again, we find that the win rate is high for $i(1)$, at 95.4%.

Online Appendix II Graphical Analysis of Bids by Region, Project Type, and Time and of Homogenized bids

In this Appendix, we show that the shapes of the distributions of Δ_{12}^2 and Δ_{23}^2 in Figure 1 remain the same when we condition on various auction characteristics such as region, project type, and year. We also show that homogenizing the bids (See Haile, Hong and Shum 2006) yields similar results. Lastly, we show that the shapes of the distributions do not depend on whether or not we normalize the bids by the reserve price. Note that for Figures OA.1 through OA.3 and for Figure OA.6, we set ε equal to 5%, i.e., we restrict the sample of auctions to those in which $b_{i(2)}^1 - b_{i(1)}^1 < 5\%$ (left panels) or $b_{i(3)}^1 - b_{i(2)}^1 < 5\%$ (right panels).

By Region

Figure OA.1 plots the histogram of Δ_{12}^2 and Δ_{23}^2 for four of the nine regions of Japan with the largest number of auctions. The regions for which we show the results are Hokkaido, Kanto, Kansai and Chubu, in decreasing order of number of total auctions.

By Project Type

In Figure OA.2, we plot the histogram of Δ_{12}^2 and Δ_{23}^2 for the four types of projects with the largest number of auctions. The four types of projects are civil engineering, repair and maintenance, paving, and communication equipment, in decreasing order of number of total auctions.

By Year

In Figure OA.3, we plot the histogram of Δ_{12}^2 and Δ_{23}^2 by year.

By Number of Bidders

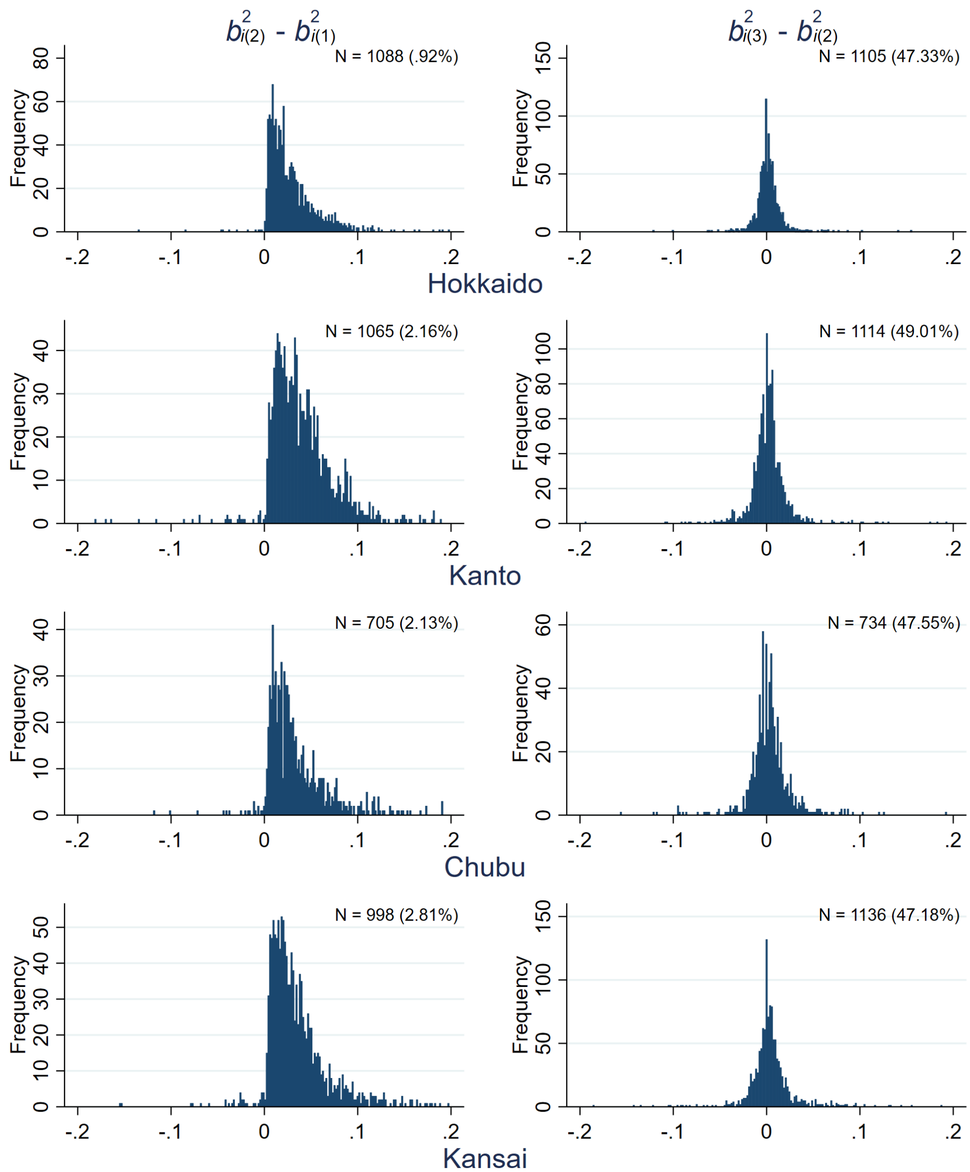
In Figure OA.4, we plot the histogram of Δ_{12}^2 and Δ_{23}^2 conditioning on the number of bidders in the auction. The panels in the top row plot Δ_{12}^2 and Δ_{23}^2 for the set of auctions in which the number of bidders is equal to or less than 9. The middle row plots auctions in which the number of bidders is equal to 10. The bottom row plots auctions with 11 or more bidders.

Homogenized Bids

In order to show that the shapes of the distributions of Δ_{12}^2 and Δ_{23}^2 remain the same even when we simultaneously control for many observed characteristics, we homogenize the bids by regressing the second-round (normalized) bid of bidder i in auction t on auction characteristics as follows (See Haile, Hong and Shum 2003):

$$b_{it}^2 = \beta_0 + \beta_1 Reserve_t + \beta_2 Reserve_t^2 + \delta_t^{Nb} + \delta_t^{ProjectType} + \delta_t^{Region-Month} + \varepsilon_{it}, \quad \text{(OA-1)}$$

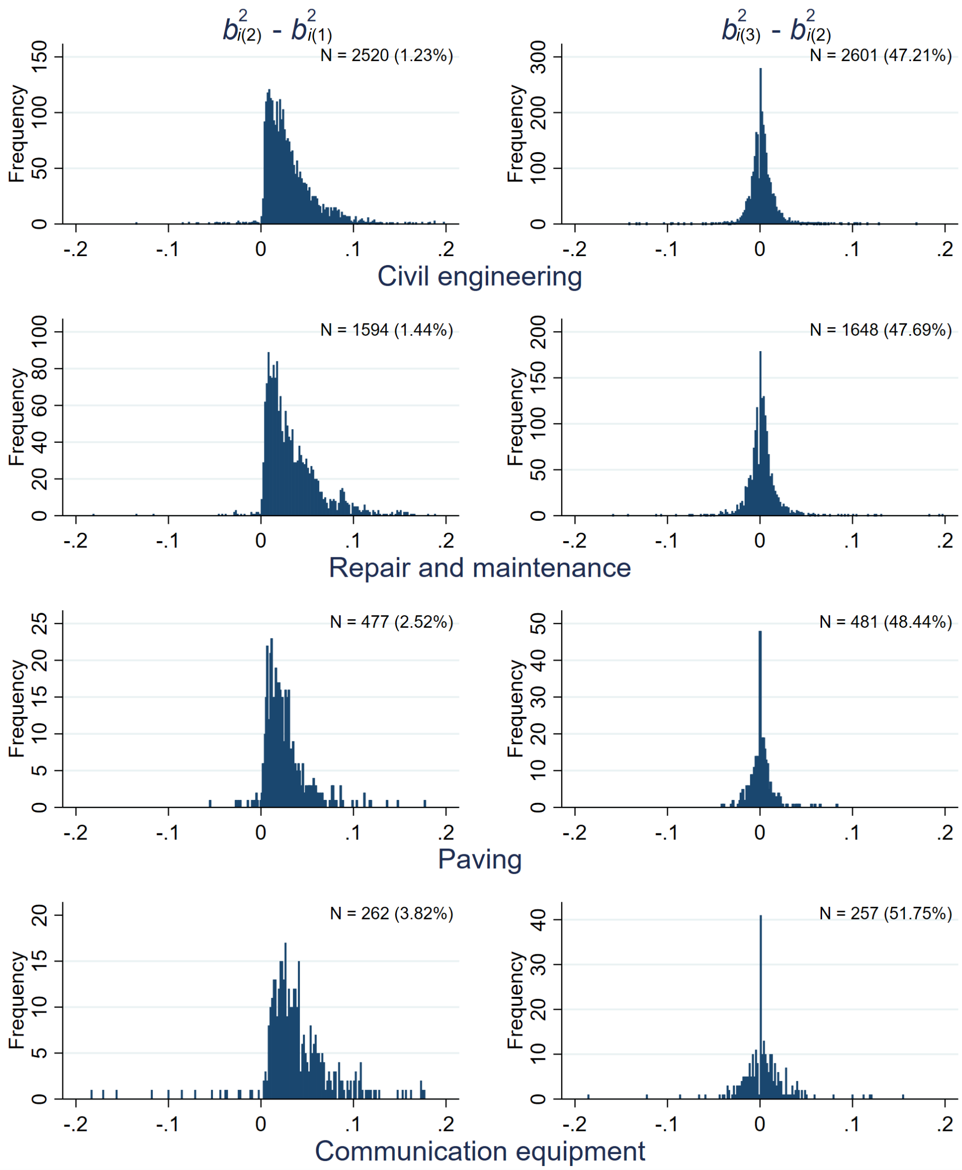
where $Reserve$ is the reserve price, δ^{Nb} is a vector of dummies that correspond to the number of bidders, $\delta^{ProjectType}$ is a vector of project types, and $\delta^{Region-Month}$ is a vector of region-month dummies. We then take the residuals from the regression and define $\tilde{\Delta}_{12}^2$ as



The left panels plot Δ^2_{12} for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot Δ^2_{23} for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

Figure OA.1: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Region.

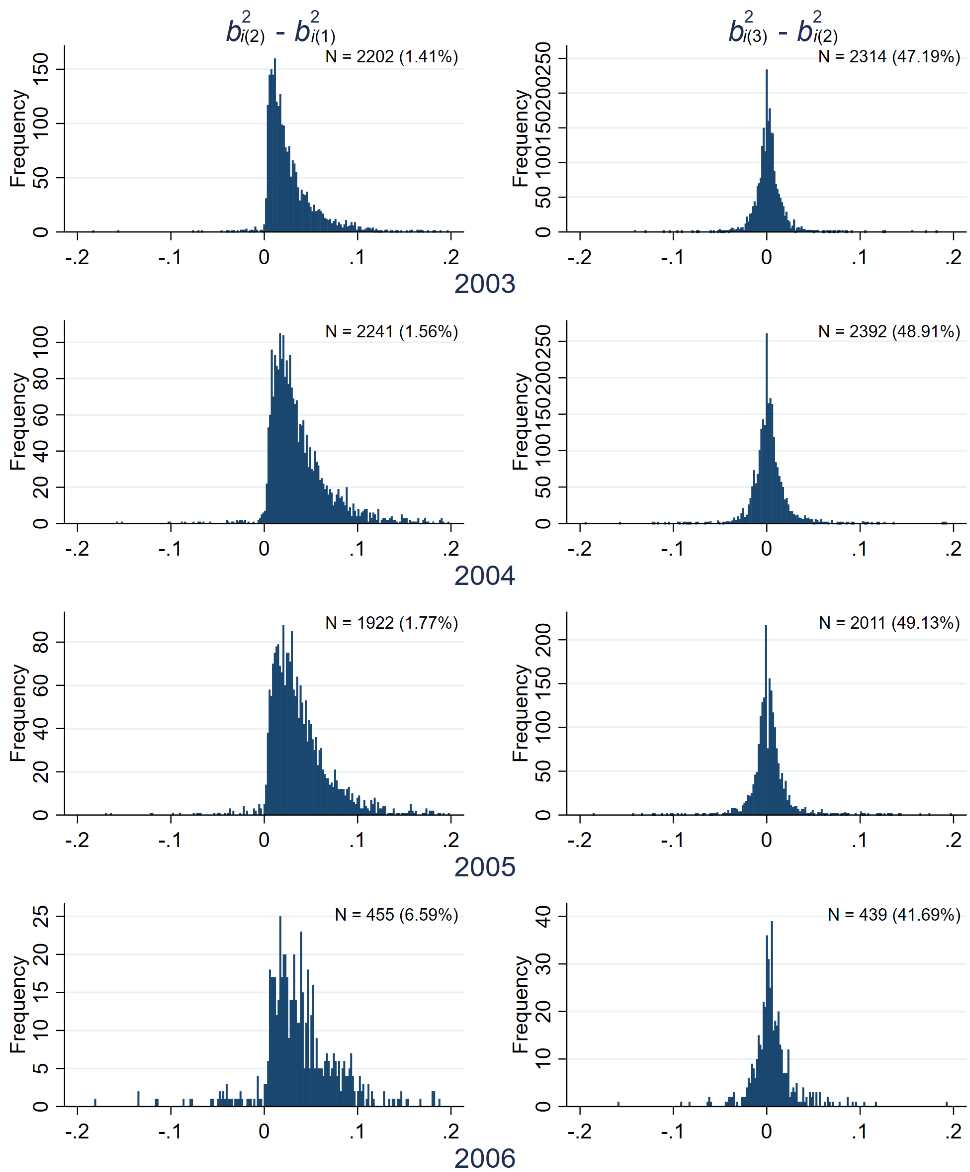
$\tilde{\Delta}^2_{12} = \varepsilon_{i(2)t} - \varepsilon_{i(1)t}$ and $\tilde{\Delta}^2_{23}$ as $\tilde{\Delta}^2_{23} = \varepsilon_{i(3)t} - \varepsilon_{i(2)t}$. Because we do not include any bidder specific covariates in the regression, the sign of $b^2_{it} - b^2_{jt}$ is the same as the sign of $\varepsilon^2_{it} - \varepsilon^2_{jt}$. This implies that Δ^2_{12} and Δ^2_{23} have the same sign as $\tilde{\Delta}^2_{12}$ and $\tilde{\Delta}^2_{23}$. Figure OA.5 plots the



The left panels plot Δ_{12} for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot Δ_{23} for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

Figure OA.2: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Project Type.

distribution of $\tilde{\Delta}^2_{12}$ and $\tilde{\Delta}^2_{23}$.

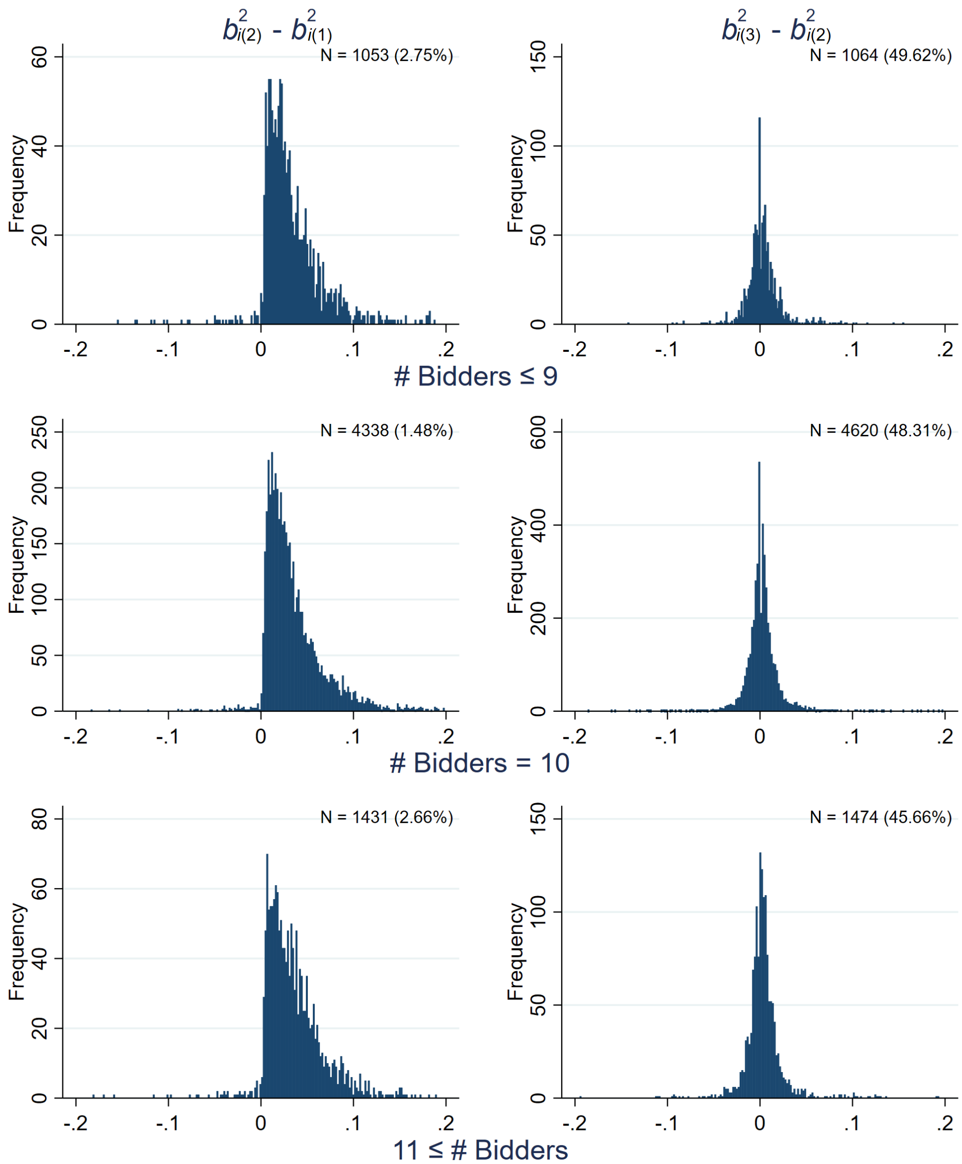


The left panels plot Δ_{12} for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot Δ_{23} for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

Figure OA.3: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by Year.

Raw Bids

In Figure OA.6, we plot the raw difference in the second-round bids without normalizing the bids by the reserve price. The left panels plot the second-round bid differences of $i(1)$



The left panels plot Δ_{12} for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5%. The right panels plot Δ_{23} for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5%.

Figure OA.4: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panel) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panel), by number of bidders.

and $i(2)$. The right panels plot the second-round bid differences of $i(2)$ and $i(3)$. The top panels correspond to auctions whose reserve price is between 20 and 22 million yen. The middle and bottom panels correspond to auctions with a reserve price between 60 and 66

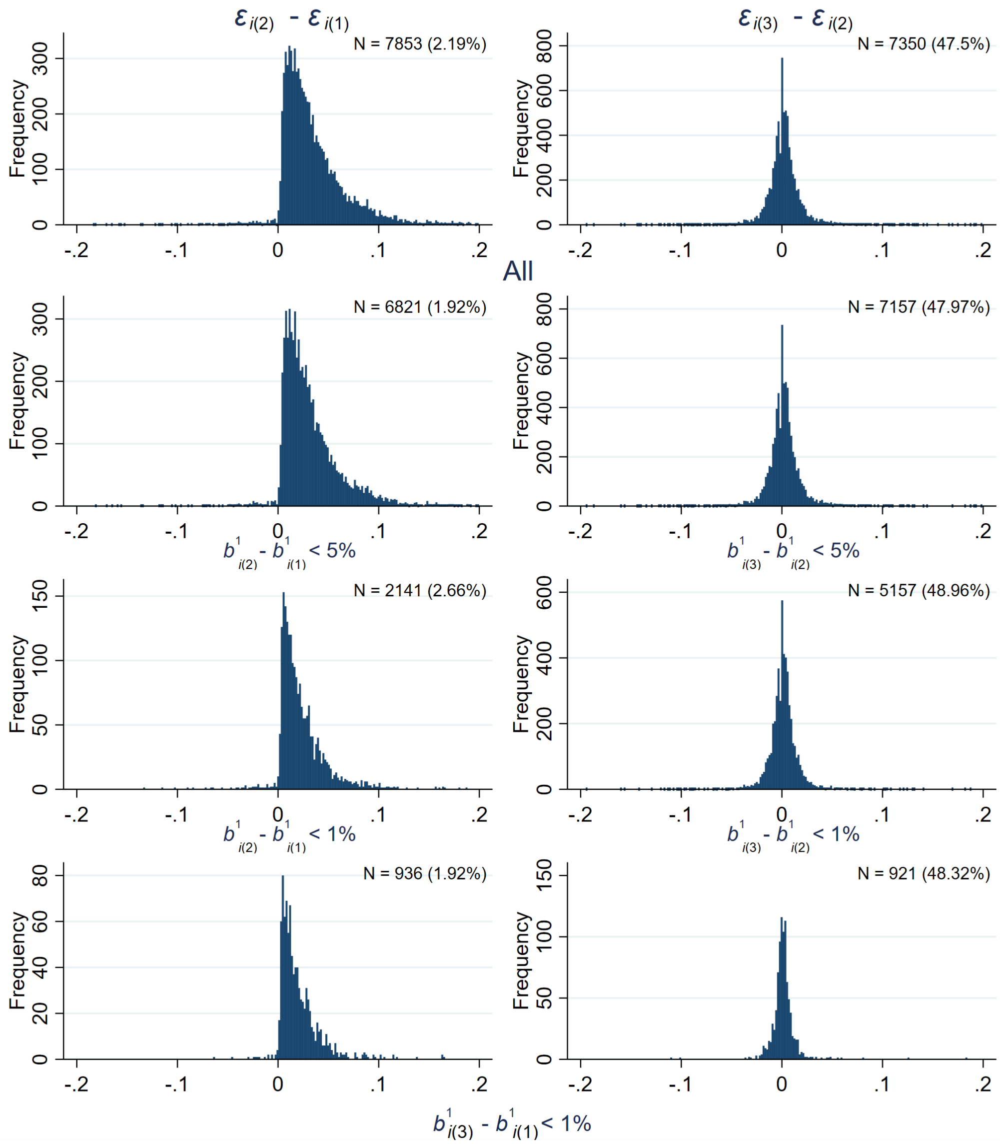
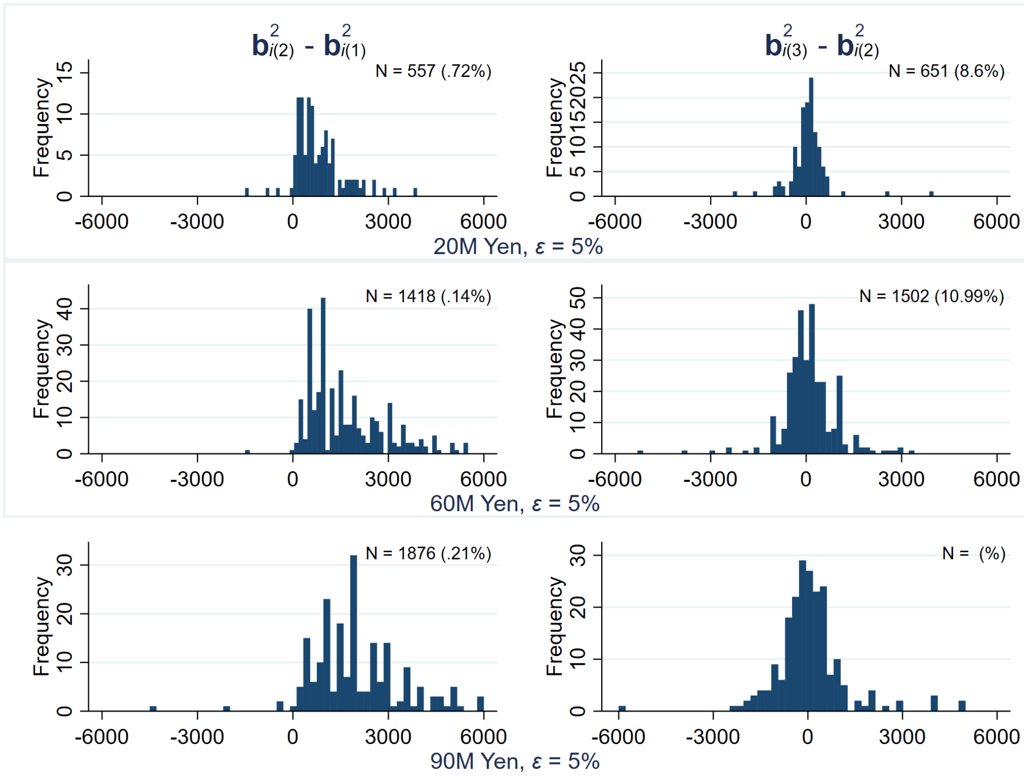


Figure OA.5: Difference in the Residuals of the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Residuals of the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels).

million yen and 90 and 99 million yen, respectively.[59] The auctions in each row roughly correspond to the 25%, 50% and 75% quantiles in terms of project size.

[59]The length of the bandwidth we use (i.e., 2 million, 6 million, and 9 million yen, respectively) is roughly 10% of the average reserve price.



The left panels plot the raw difference in bids for the set of auctions in which the first-round bids of $i(1)$ and $i(2)$ are within 5% of the reserve price. The right panels plot the raw difference in bids for the set of auctions in which the first-round bids of $i(2)$ and $i(3)$ are within 5% of the reserve price.

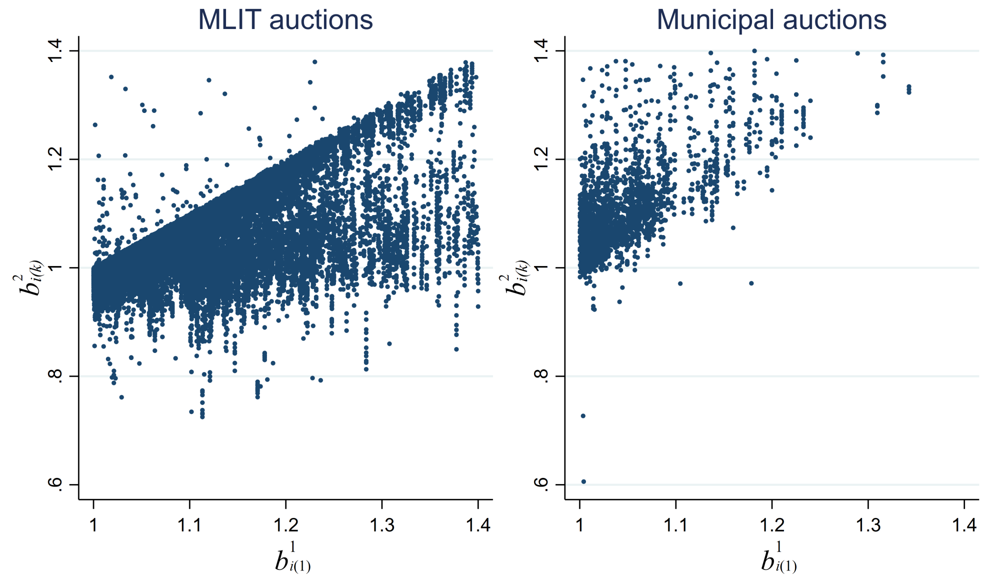
Figure OA.6: Raw Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Raw Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels).

Online Appendix III Analysis of Municipal Auctions

In order to examine whether the announcement of the lowest bid can explain the observed bidding patterns, we collect additional bidding data from three municipalities. There is significant overlap between the participants of the municipal auctions and the participants of the auctions in the baseline sample.[60] The format of the municipal auctions is very similar to that of the MLIT auctions with one key difference: in the municipal auctions, none of the bids are announced at the end of each round.

The sample statistics of municipal auctions that we use are given in Table OA.3. We report the reserve price of the auction (Column (1)), the winning bid (Column (2)), the ratio of the winning bid to the reserve price (Column (3)), the lowest bid in each round as a

[60] About a third of the bidders in the municipal auctions also bid on the MLIT auctions.



The left panel is a plot of $(b^1_{i(1)},\ b^2_{i(k)})$ for MLIT auctions and the right panel is for municipal auctions. Note that $b^1_{i(1)}$ is always larger than 1 because we condition on auctions that reach the second round. In the left panel, $b^2_{i(k)}$ is almost always less than $b^1_{i(1)}$ reflecting the fact that $b^1_{i(1)}$ is announced to all the bidders.

Figure OA.7: Plot of $b^1_{i(1)}$ and $b^2_{i(k)}$ ($k \geqq 2$) for Auctions that Reach the Second Round.

percentage of the reserve price (Columns (4)-(6)), and the number of bidders (Column (7)). The sample statistics are reported separately by whether the auction concludes in Round 1, Round 2, or Round 3.

In order to highlight the fact that, unlike in the MLIT auctions, the lowest bid is not announced in the municipal auctions, Figure OA.7 plots $(b^1_{i(1)}, b^2_{i(k)}; k \geqq 2)$, i.e., the relationship between the *first-round bid* of $i(1)$ and the *second-round bid* of $i(k)$ for MLIT auctions (left panel) and for municipal auctions (right panel) conditional on auctions that reach the second round. We find that $b^2_{i(k)}$ is almost always below $b^1_{i(1)}$ in the left panel, reflecting the fact that $i(k)$ knows $i(1)$'s first-round bid in the MLIT auctions. In the right panel, we find that there are many instances in which $b^2_{i(k)}$ is above $b^1_{i(1)}$.

Despite the fact that the lowest bid is not revealed after the first auction in municipal auctions, the bidding pattern in the second round of the municipal auctions exhibit features that are very similar to that of MLIT auctions. Figure OA.8 replicates Figure 1 for municipal auctions. Figure OA.8 shows that there is a kink in the distribution of Δ^2_{12} at zero. The fact that the bidding pattern in the MLIT and the municipal auctions exhibit similar features suggests that the announcement of the lowest bid is unlikely to be the reason for

Concluding Round	(R)eserve Yen M.	(W)inbid Yen M.	(W)/(R)	Lowest bid / Reserve			# Bidders	N
				Round 1	Round 2	Round 3		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	20.233	18.278	0.91	0.91	-	-	9.623	2,361
	(64.05)	(59.31)	(0.117)	(0.117)			(4.62)	
2	12.516	12.321	0.979	1.024	0.979	-	9.92	276
	(21.76)	(21.59)	(0.037)	(0.042)	(0.037)		(4.81)	
3	16.533	16.401	0.985	1.079	1.043	0.985	10.59	211
	(41.70)	(41.48)	(0.02)	(0.071)	(0.048)	(0.02)	(4.86)	
Total	19.211	17.562	0.922	0.934	1.007	0.985	9.72	2,848
	(59.83)	(55.59)	(0.11)	(0.121)	(0.053)	(0.02)	(4.66)	

Note: The first row corresponds to the summary statistics of auctions that ends in the first round; the second row corresponds to auctions that ends in the second round; and the third row corresponds to auctions that proceeds to the third round. The last row reports the summary statistics of all auctions. Standard deviations are reported in parentheses. First and second columns are in millions of yen.

Table OA.3: Sample Statistics for Municipal Auctions

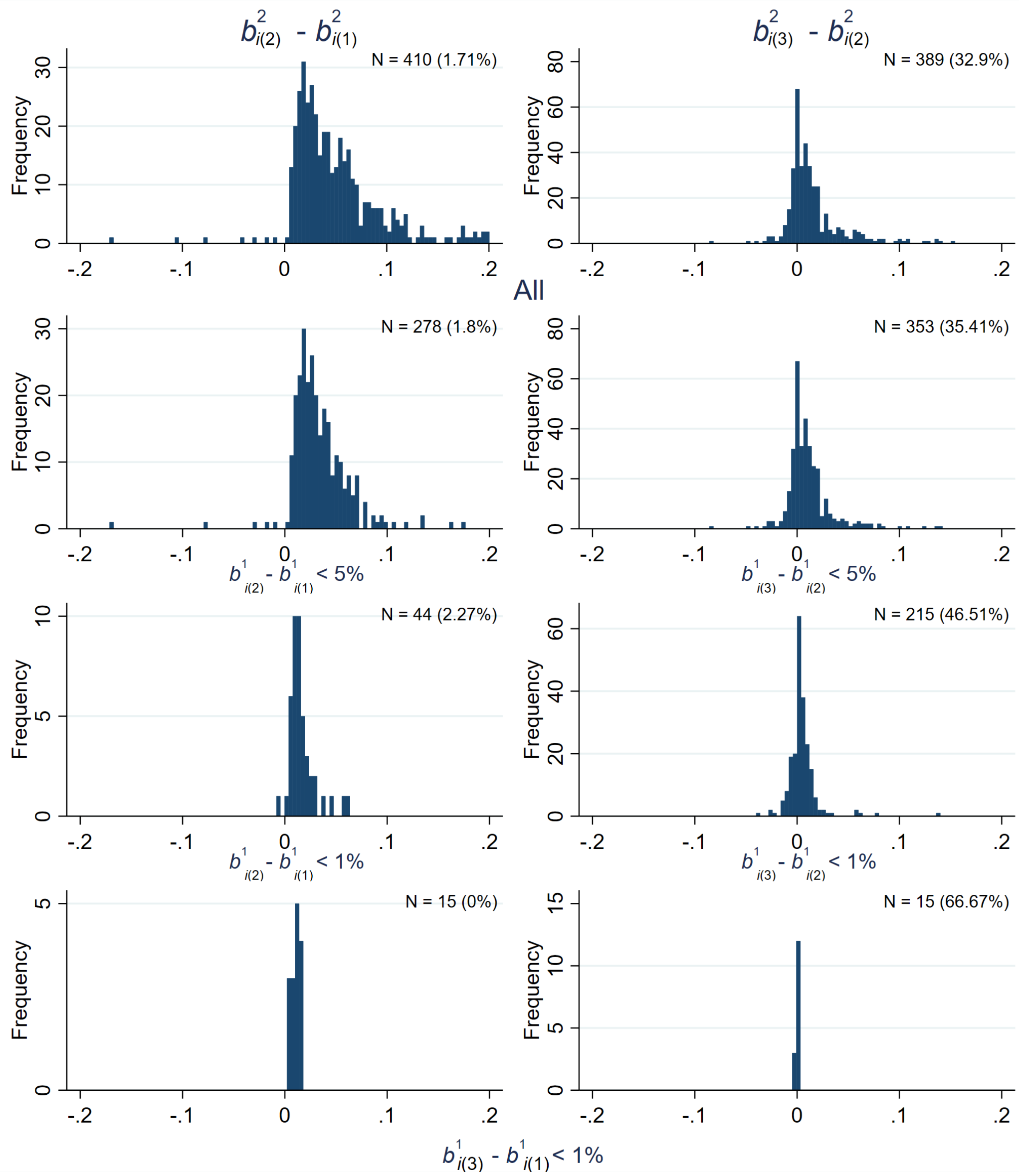
the observed bidding pattern.

Online Appendix IV Case Study

In this section, we analyze four collusion cases that were implicated by the JFTC during our sample period. The four cases are the bidding rings of: (A) prestressed concrete providers; (B) firms installing traffic signs; (C) builders of bridge upper structures; and (D) floodgate builders.[61] In all of these cases, firms were found to have engaged in activities such as deciding on a predetermined winner for each project and communicating among the members about how each bidder will bid.[62] All of the implicated firms in cases (B), (C) and (D) admitted wrongdoing soon after the start of the investigation, but none of the firms

[61] See JFTC Recommendation #27-28 (2004) and Ruling #26-27 (2010a) for case (A); JFTC Recommendation and Ruling #5-8 (2005b) for case (B); JFTC Recommendation and Ruling #12 (2005a) for case (C); and JFTC Cease and Desist Order #2-5 (2007) for case (D).

[62] In all of these cases, the ring members took turns being the predetermined winner. The determination of who would be the predetermined winner depended on factors such as whether a given firm had an existing project that was closely related to the one being auctioned and the number of auctions a given firm had won in the recent past.



This figure is the analogue of Figure 1 for municipal auctions. The figure plots Δ^2_{12} in the left panels and Δ^2_{23} in the right panels.

Figure OA.8: Difference in the Second-Round Bids for Municipal Auctions.

implicated in case (A) admitted any wrongdoing initially, and the case went to trial.[63]

Before we analyze these four cases, we point out one interesting feature of the bidding ring in case (A): According to the ruling in case (A), an internal rule existed among the

[63]Out of 20 firms that were initially implicated in case (A), one firm was acquired by another firm, one was acquitted, and the rest of the firms eventually settled with the JFTC after going to trial.

subset of the ring members operating in the Kansai region, which prescribed that: 1) the predetermined winner should aim to bid below the reserve price in the first round; 2) if the predetermined winner did not bid below the reserve price in the first round, the firm should submit a second-round bid that is less than some prespecified fraction (e.g., 0.97) of its first-round bid (i.e., $b^2_{i(1)} < 0.97 \times b^1_{i(1)}$); and 3) the rest of the ring members should submit second-round bids that are higher than the prespecified fraction of the *predetermined winner's* first-round bid (e.g., $b^2_{i(k)} > 0.97 \times b^1_{i(1)}$ for $k \neq 1$). The prespecified fractions used in the ring were 0.96 for auctions with an expected value less than 100 million yen; 0.97 for auctions with an expected value between 100 million yen and 500 million yen; and 0.975 for auctions expected to be worth more than 500 million yen. One consequence of this internal rule is that the lowest cartel bidder will be the same in Round 1 and Round 2.

Figure OA.9 is a time series plot of the winning bid of auctions in which the winner is a member of one of the implicated bidding rings. We have also drawn a vertical line that corresponds to the "end date" of collusion. The "end date" is the date, according to the JFTC's ruling, after which the ring members were deemed to have stopped colluding. The date roughly corresponds to the start date of the investigation. Note that in panels (B) and (C) of Figure OA.9, there exist periods after the collusion end date during which no ring member wins an auction. This reflects the fact that implicated ring members in cases (B) and (C) were banned from participating in public procurement projects for a period of up to 18 months.[64]

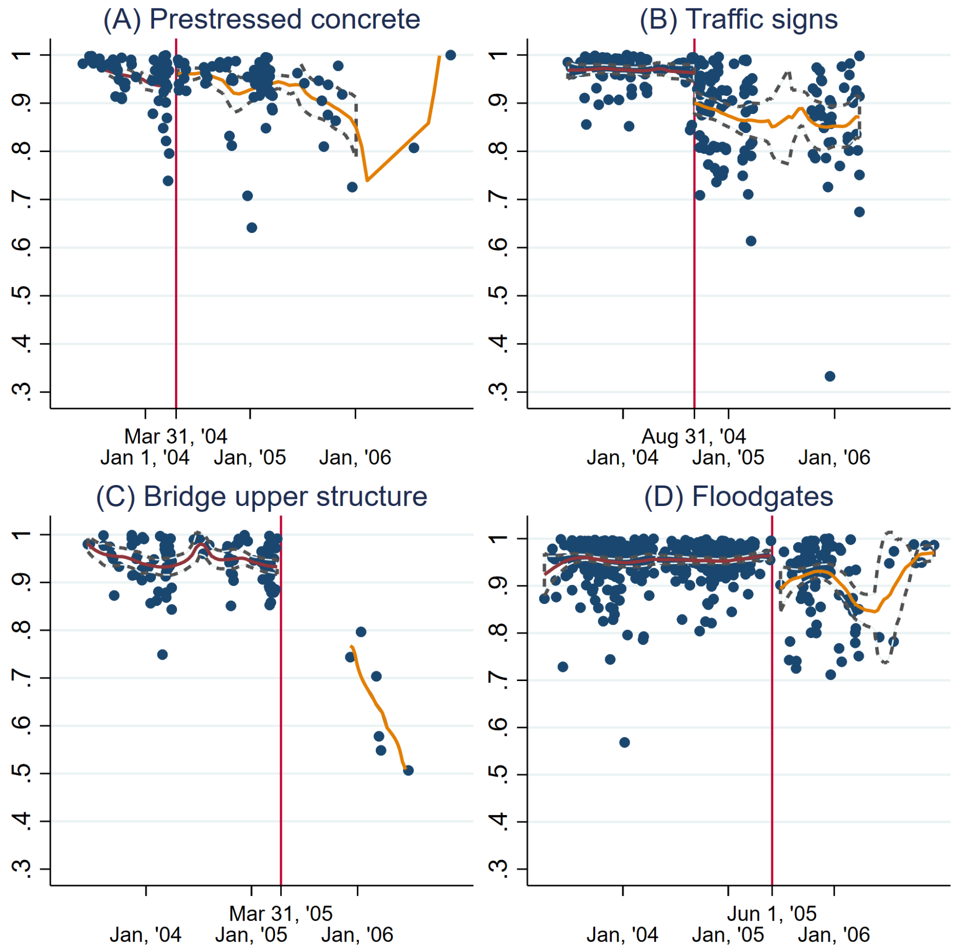
Figure OA.9 shows that for cases (B), (C), and (D), there is a general drop in the winning bid of about 8.3%, 19.5%, and 5.3%, respectively, after the collusion end date. However, there is almost no change in the winning bid for case (A) before and after the end date. Also, it is worth mentioning that, even for cases (B), (C), and (D), there are some auctions in which the winning bid is extremely high after the end date.[65] While the investigation and the ruling of the JFTC seem to have made collusion harder, it is far from clear whether the prices after the end date are truly at competitive levels. We revisit this point below.

We now examine the second-round bids of $i(1)$, $i(2)$, and $i(3)$ during the period in which the firms were colluding.[66] If the distinctive shapes of the distribution of Δ^2_{12} and Δ^2_{23} that we found in Section 4 are, indeed, evidence of collusion, we should expect to see

[64]The ring members involved in cases (A) and (D) were banned from bidding in procurement auctions in 2010 and 2007, respectively, which are after the sample period.

[65]In fact, about 24.4% of auctions after the end date have a winning bid higher than 95% for cases (B), (C) and (D).

[66]Note that $i(1)$, $i(2)$, and $i(3)$ are not necessarily members of the ring if an outsider bids in the auction.

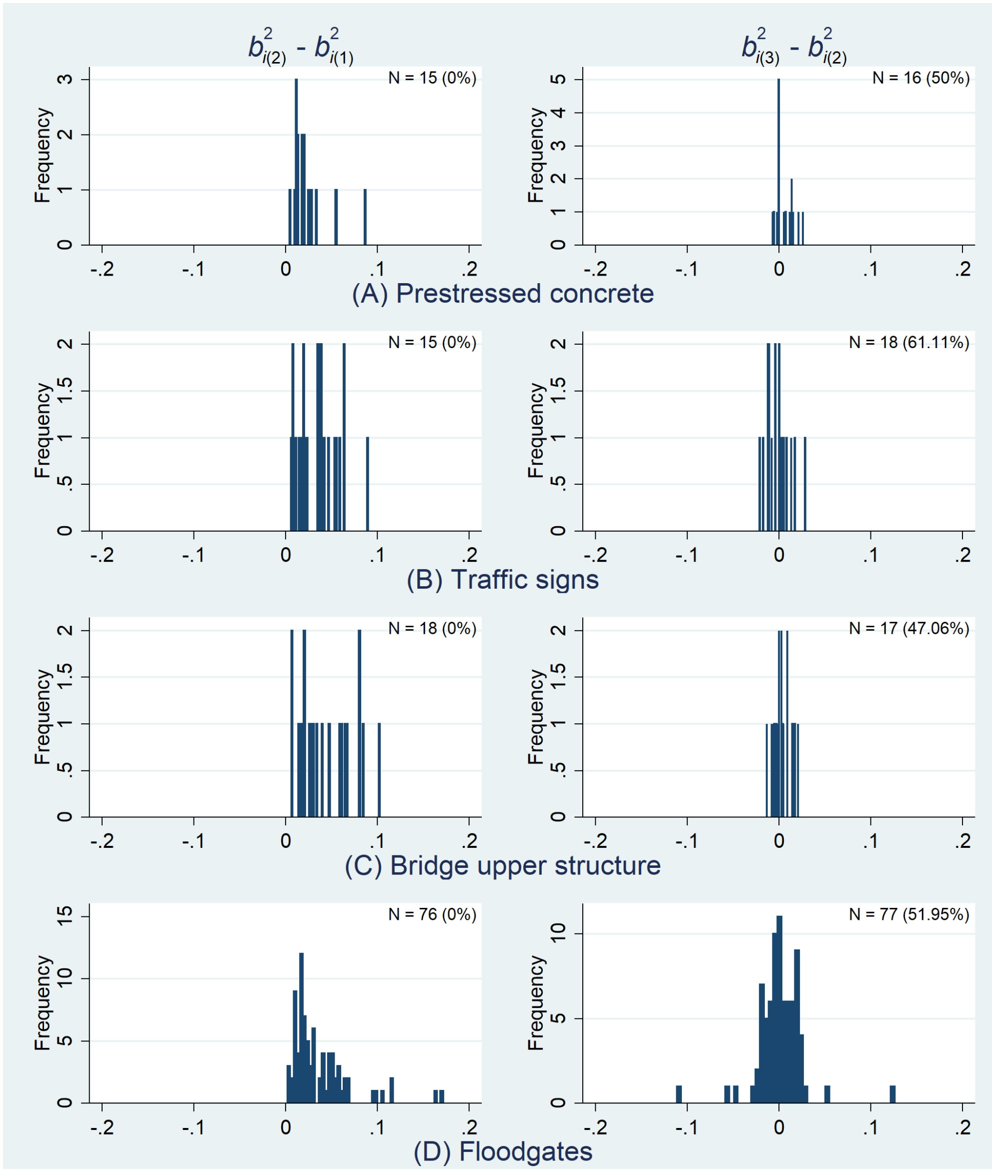


The horizontal axis corresponds to the calendar date from the beginning of our sample (i.e., April 1, 2003), and the vertical axis corresponds to the winning bid as a percentage of the reserve price. The vertical line in each of the four panels corresponds to the collusion "end date."

Figure OA.9: Winning Bids of Cartel Members.

the same pattern among the second-round bids of these colluding firms. Figure OA.10 plots the histogram of Δ^2_{12} and Δ^2_{23} before the collusion end date for auctions won by each of the four bidding rings. The samples used for the figure correspond to the set of auctions in which $b^1_{i(2)} - b^1_{i(1)} < 5\%$ for the left column and $b^1_{i(3)} - b^1_{i(2)} < 5\%$ for the right column. We see that for all four bidding rings, the histogram of Δ^2_{12} is asymmetric around zero, while the histogram of Δ^2_{23} is symmetric around zero. Thus, Figure OA.10 suggests that the distinctive shapes of the distributions of Δ^2_{12} and Δ^2_{23} are a hallmark of collusive bidding.

We next examine the second-round bids of the ring members, but for auctions occurring



The left panels correspond to auctions in which $b^1_{i(2)} - b^1_{i(1)} < 5\%$, and the right panels correspond to auctions in which $b^1_{i(3)} - b^1_{i(2)} < 5\%$. N is the sample size, and the number in the parenthesis corresponds to the fraction of auctions that lie to the left of zero.

Figure OA.10: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels) Before the Collusion End Date.

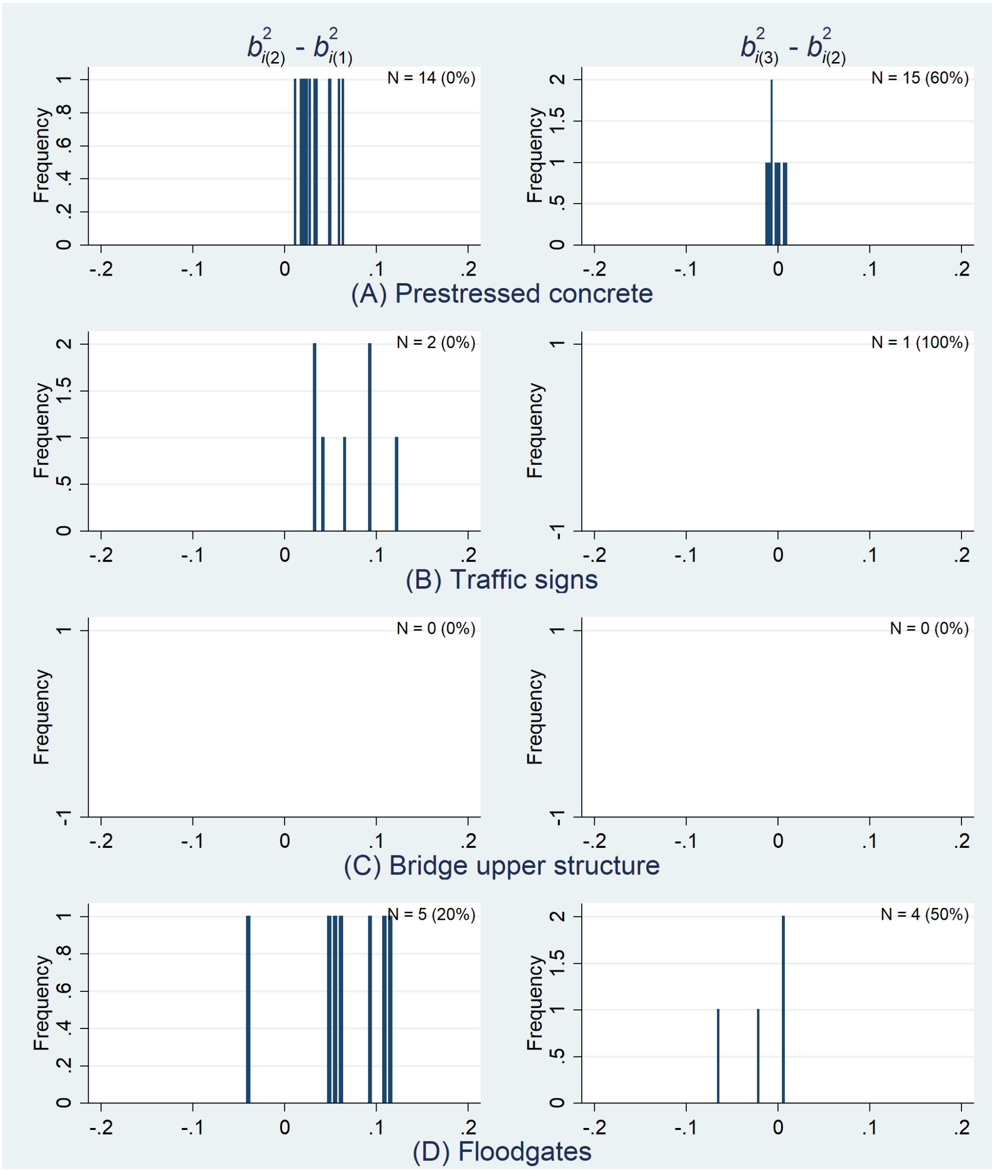
after the collusion end date. To the extent that ring members stopped colluding after the end date, we should expect to see Δ^2_{12} to lie to the left of zero in a fair number of auctions. Figure OA.11 plots the histogram of Δ^2_{12} and Δ^2_{23} for each of the four bidding rings with

$b^1_{i(2)} - b^1_{i(1)} < 5\%$ and $b^1_{i(3)} - b^1_{i(2)} < 5\%$. Although the sample sizes are very small, the distributions of Δ^2_{12} and Δ^2_{23} in Figure OA.11 are similar to those in Figure OA.10. That is, Δ^2_{12} is distributed to the right of zero, while Δ^2_{23} is distributed symmetrically around zero. This may seem to cast doubt on our analysis – why do the distinctive patterns in the distribution of Δ^2_{12} and Δ^2_{23} persist even after the collusion end date, when firms presumably started behaving competitively?

Our view is that the asymmetry in the distribution of Δ^2_{12} should be taken as evidence that the implicated firms were able to continue colluding at least on some auctions, even after the end date. While the bidding rings seem to have changed their behavior around the time of the end date – as the drop in the winning bid suggests in Figure OA.9 – this does not necessarily mean that the firms completely ceased to collude. For example, a number of firms implicated in case (C) were also subsequently charged and found guilty of collusion in a separate case by the JFTC.

With respect to case (A), there is additional evidence that the ring members continued to collude beyond the end date by following the formula for rebids that we described earlier. Recall that a subset of the prestressed concrete ring members in the Kansai region had a prespecified discount (0.96 for auctions valued at less than 100 million yen; 0.97 for auctions valued between 100 million yen and 500 million yen; and 0.975 for auctions valued at more than 500 million yen) that they used when rebidding in the second round. Figure OA.12 plots the second-round bids of the ring members in the Kansai region as a fraction of the lowest first-round bid. The top panel corresponds to auctions with a reserve price below 100 million yen; the middle corresponds to those with a reserve price between 100 and 500 million yen; and the last panel corresponds to those with a reserve price of more than 500 million yen. The horizontal axis in the figure corresponds to the calendar date. The vertical line in each panel corresponds to the collusion end date. Thus, auctions that took place before the end date appear to the left of this line. The circles in the figure represent $b^2_{i(1)}/b^1_{i(1)}$, and the Xs in the figure represent $b^2_{i(k)}/b^1_{i(1)}$ for $k \neq 1$. We have drawn a horizontal line at 0.96 (top panel), 0.97 (middle panel), and 0.975 (bottom panel).

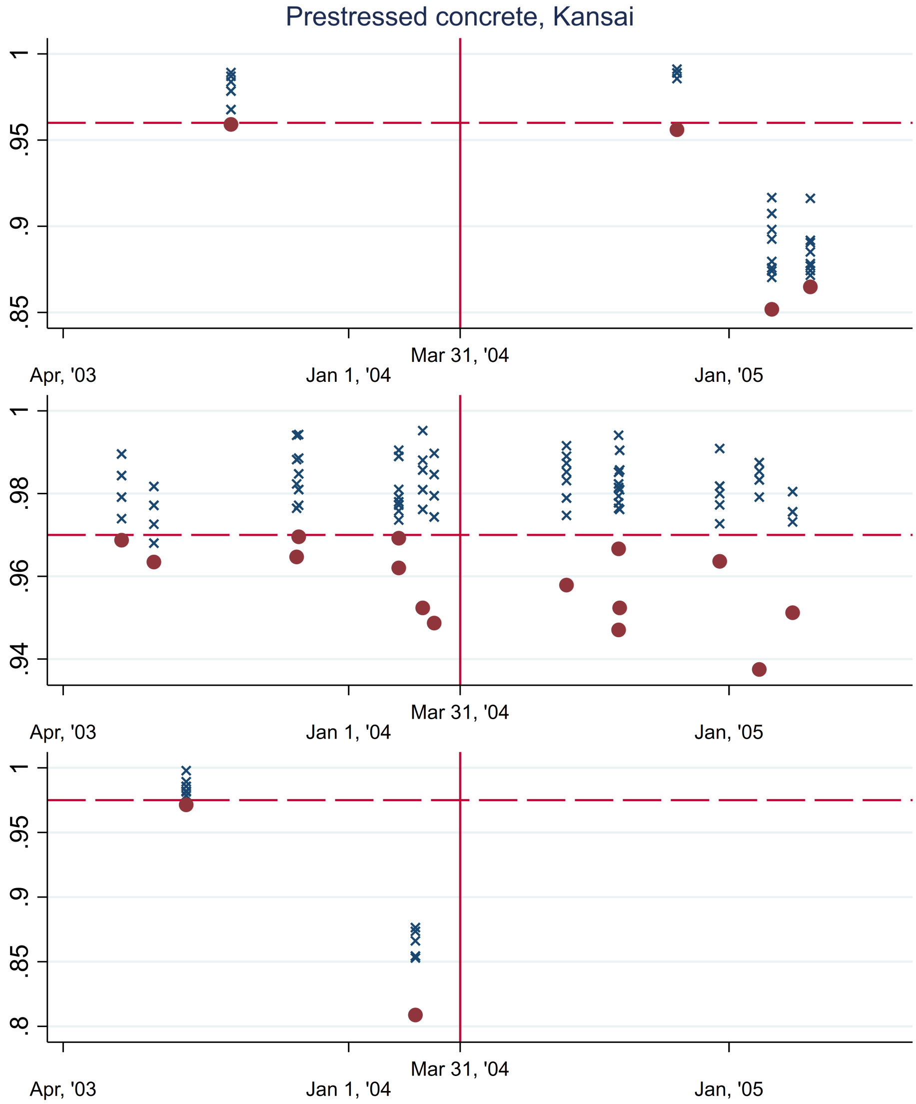
While the top and the bottom panels are not very informative, note that all of $i(1)$'s second-round bids in the middle panel of Figure OA.12 are below 0.97 of $i(1)$'s first-round bid. Moreover, the bids of all of the others are above 0.97 of $i(1)$'s first-round bid, except for one auction. If we focus on auctions after the collusion end date, the second-round bids of $i(k)$ ($k \neq 1$) are all above 0.97. The bidding pattern in Figure OA.12 suggests that bidders continued to use the prespecified discount as the threshold value for submitting



The left panels correspond to auctions in which $b^1_{i(2)} - b^1_{i(1)} < 5\%$, and the right panels correspond to auctions in which $b^1_{i(3)} - b^1_{i(2)} < 5\%$.

Figure OA.11: Difference in the Second-Round Bids of $i(1)$ and $i(2)$ (Left Panels) and the Difference in the Second-Round Bids of $i(2)$ and $i(3)$ (Right Panels) After the Collusion End Date.

second-round bids. It seems quite likely that the ring members were able to maintain some level of collusion even after the end date.



The top panel corresponds to auctions with a reserve price less than 100 million yen; the second panel corresponds to auctions with a reserve price between 100 million and 500 million yen; and the last panel corresponds to auctions with a reserve price above 500 million yen. The horizontal axis corresponds to the calendar date, starting from April 1, 2003. The vertical line at March 31, 2004 corresponds to the collusion end date of for case (A). Circles in the figure correspond to $b^2_{i(1)}/b^1_{i(1)}$, and Xs in the figure correspond to $b^2_i(k)/b^1_{i(1)}$ for $k \neq 1$.

Figure OA.12: Second-Round Bids of the Ring Members of Kansai Region as a Fraction of the Lowest First-Round Bid.

Online Appendix V Proof that $b_i^r - b_{i(1)}^r$ admits continuous, strictly positive and bounded density.

Consider the distribution of b_i^r, $f_{b_i^r}$:

$$f_{b_i^r}(t) = \int \frac{\partial (b_i^r)^{-1}(t; c_i, h_i^r)}{\partial t} f_{x_i^r|c_i}((b_i^r)^{-1}(t; c_i, h_i^r)|c_i) f_{c_i}(c_i) dc_i,$$

where $(b_i^r)^{-1}$ is the inverse of b_i^r with respect to the first element, and $f_{x_i^r|c_i}$ and f_{c_i} are the conditional density of x_i^r, and the pdf of c_i. Given that $\frac{\partial b_i^r}{\partial x_i^r}$ is continuous and bounded away from zero (Assumption 2), $\frac{\partial (b_i^r)^{-1}}{\partial t}$ is continuous and bounded. Because $f_{x_i^r|c_{i,}}$ and f_{c_i} are also continuous and bounded (Assumption 3), $f_{b_i^r}(t)$ is continuous, by dominated convergence. Note also that $f_{b_i^r}(t)$ is bounded and positive.[67] Similarly, the conditional distribution of b_i^r given $b_{i(1)}^r$ is as follows:

$$f_{b_i^r|b_{i(1)}^r}(t) = \int \frac{\partial (b_i^r)^{-1}(t; c_i; h_i^r)}{\partial t} f_{x_i^r|c_i}((b_i^r)^{-1}(t; c_i, h_i^r)|c_i) dF_{c_i|b_{i(1)}^r}.$$

Note that Assumption 1 (i.e., the independence of $x_{i,1}^r$ with respect to $\mathbf{x}_{i(1)}^r$) implies that the distribution of $f_{x_{i,1}^r|\mathbf{x}_{i,-1}^r}$ does not change. Following a similar argument as before, $f_{b_i^r|b_{i(1)}^r}$ is continuous, bounded and positive.

Now consider the distribution of $b_i^r - b_{i(1)}^r$:

$$f_{b_i^r - b_{i(1)}^r}(t) = \int f_{b_i^r|b_{i(1)}^r}(s + t|s) dF_{b_{i(1)}^r}(s).$$

Given that $f_{b_i^r|b_{i(1)}^r}(\cdot|s)$ is continuous and bounded, $f_{b_i^r - b_{i(1)}^r}$ is also continuous, by dominated convergence. It is also bounded and positive.

Online Appendix VI A Theory of Two-stage Auctions with a Secret Reserve Price

We consider an auction game with n bidders, two cost types, and two rounds. Assume that the cost type, c, is either 0 or 1, with probability θ and $1 - \theta$, i.e., $\Pr(c = 0) = \theta$, $\Pr(c = 1) = 1 - \theta$. We consider the case $0 < \theta < 1 - \sqrt[n-1]{1/3}$. Let the distribution of the reserve price in the first round be uniform [0,1]. We assume that the seller accepts any bid in the second round with ties in the second round broken in favor of low cost types.[68] Assume that the lowest bid in the first round is announced, but none of the other bids are.

[67] The fact that $f_{b_j^r}$ is positive follows from Assumption 4 and the fact that $f_{x_i^r|c_i} > 0$ (Assumption 3).

[68] More precisely, if a low cost type and a high cost type bid the same amount in the second round, we assume that the low cost type wins the auction. Tie-breaking rules for other cases (e.g., ties between two low cost types) turn out not to be important. Tie breaking rules for the first round will also not be important.

Proposition OA *There exists a Bayesian Nash equilibrium of the game. In particular, there exists an equilibrium in which (1) the high-cost type bids 1 in each round; (2) the low-cost type bids mixed strategies in both rounds; and (3) the first-round bidding strategy of the low-cost type,* $H(\cdot)$*, is a c.d.f over support* $[1/2+(1-\theta)^{n-1}/2, 1]$ *that is differentiable in the interior of the support, has mass at* 1*, and has no mass anywhere else.*

Proof. We consider the case of $n = 2$ because the proof for $n > 2$ is almost identical. First, consider the second round of the auction following an initial round in which the two bidders submit different bids (i.e., no tie in the first round). Let μ denote $i(1)$'s belief that $i(2)$ is a low cost type. μ is a function of $i(1)$'s first-round bid, $b^1_{i(1)}$, but we suppress this dependence for the time being. In the equilibrium that we consider, $i(1)$ is always a low cost bidder, which implies that $i(2)$'s equilibrium beliefs regarding $i(1)$'s type is degenerate, with all the mass on low cost. Moreover, μ will be strictly between 0 and 1. Consider the following mixed strategy, $F_1(\tau)$, for the low-cost $i(1)$ bidder in the second round:

$$F_1(\tau) = \begin{cases} 0 & \text{if } \tau < 1-\mu, \\ \dfrac{\tau+\mu-1}{\tau} & \text{if } \tau \in [1-\mu, 1), \\ 1 & \text{if } 1 \leqq \tau. \end{cases}$$

$F_1(\cdot)$ is a distribution with support $[1-\mu, 1]$, with a mass of $1-\mu$ at 1. Now consider the following mixed strategy, $F_2(\tau)$, for the low-cost $i(2)$ bidder:

$$F_2(\tau) = \begin{cases} 0 & \text{if } \tau < 1-\mu, \\ \dfrac{\tau+\mu-1}{\tau\mu} & \text{if } \tau \in [1-\mu, 1), \\ 1 & \text{if } 1 \leqq \tau. \end{cases}$$

For the high cost type, we consider a bidding strategy in which the bidder bids 1 with probability 1. We show below that $F_1(\cdot)$, $F_2(\cdot)$, and the bidding strategy of the high cost type constitute best responses.

Consider the second-round payoff of low-cost $i(1)$ bidder when it bids $b^2_{i(1)} = b$:

$$\pi^2_{i(1)}(b) = (1 - F_2(b))b\mu + (1-\mu)b.$$

Note that b is the profit margin, $\mu(1 - F_2(b))$ is the probability that the opponent is a low cost type and bids higher than b, and $1-\mu$ is the probability that the opponent is a high cost type. Substituting the expression for $F_2(\cdot)$, we obtain

$$\begin{aligned}\pi^2_{i(1)}(b) &= b\left[\left(1-\frac{b+\mu-1}{b\mu}\right)\mu+(1-\mu)\right], \\ &= 1-\mu.\end{aligned}$$

Hence, $\pi^2_{i(1)}(b)$ is constant for all $b\in[1-\mu,1)$. Any $b\in[1-\mu,1]$ maximizes low-cost $i(1)$'s payoffs given $i(2)$'s strategy, $F_2(\cdot)$.

Similarly, the payoff of low-cost $i(2)$ bidder when it bids $b\in[1-\mu,1)$ is as follows:

$$\begin{aligned}\pi^2_{i(2)}(b) &= b\left[(1-F_1(b))\right], \\ &= b\left[\left(1-\frac{b+\mu-1}{b}\right)\right], \\ &= 1-\mu,\end{aligned}$$

where we have used the fact that $i(2)$ believes that $i(1)$ is a low-cost type with probability 1. We find that $\pi^2_{i(2)}$ is constant for $b\in[1-\mu,1)$. Bidding any value in $[1-\mu,1)$ maximizes low-cost $i(2)$'s payoffs given $i(1)$'s strategy. It is easy to see that bidding 1 with probability 1 is also a best response for the high-cost type.[69] Hence, the strategies described above constitute best responses.

Now consider the second-round of the auction game following an initial round in which the bidders bid identically. We only consider the case in which the first-round tie occurs at bids equal to 1 because this is the only case that will occur with positive probability on the equilibrium path. In this case, we need not make a distinction between $i(1)$ and $i(2)$, because both bidders are symmetric. Letting μ denote bidders' belief that the opponent is a low-cost type, the following bidding strategy constitutes a best response for the low-cost types:

$$F(\tau)=\begin{cases} 0 & \text{if } \tau<1-\mu, \\ \dfrac{\tau+\mu-1}{\mu\tau} & \text{if } \tau\in[1-\mu,1), \\ 1 & \text{if } 1\leqq\tau. \end{cases}$$

We now consider the first round of the auction game. Recall that we consider an equi-

[69]Recall that a tie between a high cost type and a low cost type is broken in favor of the low cost type by assumption.

librium in which the low-cost types play a mixed strategy, say $H(\cdot)$, and the high-cost types play a pure strategy (with all the mass at 1). We focus on a bidding strategy $H(\cdot)$ such that $H(\cdot)$ has support between $[1-\theta/2, 1]$, is differentiable in the interior of the support, has mass at 1, and has no mass anywhere else.

The expected payoff of a low-cost bidder from bidding b, $\pi^1(b)$, is as follows:

$$
\begin{aligned}
\pi^1(b) = \quad & \overbrace{\theta \int_{t=0}^{t=b} t(1-\mu(t))dH(t)}^{\text{opponent bids lower and proceed to second round}} \\
& + \quad \overbrace{[1-\theta H(b)]\, b(1-\mu(b))}^{\text{opponent bids higher and proceed to second round}} && \text{if } b < 1 \\
& + \overbrace{[1-\theta H(b)]\,(1-b)b}^{\text{win in the first round}} \\
\pi^1(1) = \; & \overbrace{\theta \int_{0<t<1} t(1-\mu(t))dH(t)}^{\text{profit when opponent bids less than 1}} \\
& + \overbrace{(1-\theta+\theta H_\delta) \times (1-\mu(1))}^{\text{profit when opponent bids 1}},
\end{aligned}
$$

where $H_\delta = 1 - \lim_{b \to 1} H(b)$ (i.e., the mass point at 1).

The first expression corresponds to the bidder's expected profit when $b < 1$. The expression has three components, corresponding to three possible outcomes. The first term corresponds to the expected profit when the opponent bids lower and the auction proceeds to the second round. The second term corresponds to the case in which the opponent bids higher and the auction proceeds to the second round. The last term corresponds to the case in which the auction ends in the first round. The second expression corresponds to the bidder's expected profit when $b = 1$. Note that we now make the dependence of μ on the first-round bid explicit, as $\mu(b)$.

The expression for μ is given by Bayes rule as follows:

$$
\mu(t) = \begin{cases} \dfrac{\theta(1-H(t))}{\theta(1-H(t))+(1-\theta)} & \text{if } t \in [0,1), \\[2ex] \dfrac{\theta H_\delta}{\theta H_\delta + (1-\theta)} & \text{if } t = 1. \end{cases}
$$

Substituting this expression into the expression for the expected profit, we obtain the following expression:

$$\pi^1(b) = \theta \int_{t=0}^{t=b} t \frac{1-\theta}{1-\theta H(t)} dH(t) + (1-\theta)b + [1-\theta H(b)]\,(1-b)b, \qquad \text{if } b<1$$
$$\pi^1(1) = \theta \int_{0<t<1} t \frac{1-\theta}{1-\theta H(t)} dH(t) \ + (1-\theta).$$

Taking the derivative of $\pi^1(b)$ ($b<1$) we obtain the following expression:

$$\begin{aligned}\frac{\partial \pi^1(b)}{\partial b} &= \left[\theta b \frac{1-\theta}{1-\theta H(b)} - \theta(1-b)b\right] H'(b) \\ &\qquad +(1-\theta) + [1-\theta H(b)](1-2b).\end{aligned}$$

Setting this expression equal to zero, and solving for H', we obtain the following expression,

$$H'(b) = \frac{[1-\theta H(b)]\,[(1-\theta) - [1-\theta H(b)](2b-1)]}{\theta b[(1-\theta H(b))(1-b) - (1-\theta)]}. \tag{OA-2}$$

Lemma OA-1 below guarantees that expression (OA-2) with an initial condition $H(1-\theta/2) = 0$ admits a solution that is monotone increasing in range $[1-\theta/2, 1)$ and bounded above by 1. If we define $H(1) = 1$, $H(\cdot)$ will be a proper distribution function.

We wish to prove that bidding according to $H(\cdot)$ for low cost types is a best response to other low cost types bidding $H(\cdot)$ (and high cost types bidding 1 with probability 1). If other low cost types bid according to $H(\cdot)$, the payoff from bidding any b between $1-\theta/2$ and 1 (i.e., $b \in [1-\theta/2, 1)$) yields the same expected payoff to the low cost type, by construction. Below, we show that bidding exactly equal to 1 yields the same payoff as bidding just below 1 and that bidding below $1-\theta/2$ yields lower payoffs than $\pi^1(1-\theta/2)$.

First we show that bidding exactly equal to 1 yields the same payoff as bidding just below 1. Consider the payoff from bidding just below 1;

$$\pi^1(1-\varepsilon) \ = \ \theta \int_{t=0}^{1-\varepsilon} t \frac{1-\theta}{1-\theta H(t)} dH(t) + (1-\theta) + [1-\theta H(1-\varepsilon)]\,\varepsilon(1-\varepsilon).$$

Taking ε to zero, we find

$$\lim_{\varepsilon\to 0} \pi^1(1-\varepsilon) = \theta \int_{0<t<1} t \frac{1-\theta}{1-\theta H(t)} dH(t) + (1-\theta),$$

which is the same expression as $\pi^1(1)$, the payoff from bidding exactly 1. Hence, the expected payoff from bidding 1 is the same as bidding just below it.

Now consider bidding lower than $1-\theta/2$. The payoff associated with b ($b \leqq 1-\theta/2$) is as follows:

$$\begin{aligned}\pi^1(b) &= \overbrace{b(1-b)}^{\text{expected profit from first round}} + \overbrace{b(1-\mu)}^{\text{expected profit from second round}} \\ &= b(2-\theta-b).\end{aligned}$$

Given that this expression is strictly increasing for $b \leqq 1-\theta/2$, $\pi_1^1(b) < \pi_1^1(1-\theta/2)$ for $b < 1-\theta/2$. ∎

When $n > 2$, the differential equation that defines the equilibrium strategy of a low-cost type in the first round is given by

$$H'(b) = \frac{[1-\theta H(b)]\,[(1-\theta)^{n-1} - [1-\theta H(b)]^{n-1}(2b-1)]}{(n-1)\theta b[(1-\theta H(b))^{n-1}(1-b) - (1-\theta)^{n-1}]}.$$

Corollary OA *Let b be any number between 0 and 1, i.e.,* $b \in [0,1]$. *Assume that the lowest bid in the first round is b, i.e.,* $b_{i(1)}^1 = b$. *Then, there exists* $\varepsilon > 0$ *such that if* $b_{i(2)}^1 - b_{i(1)}^1 < \varepsilon$, *the probability that* $i(2)$ *outbids* $i(1)$ *in the second round is above 1/2.*

Proof. Consider the case in which $b < 1$. Then take ε so that $b + \varepsilon < 1$. When $b_{i(2)}^1 - b_{i(1)}^1 < \varepsilon$, both $i(1)$ and $i(2)$ are low cost types. Then the probability that $i(2)$ wins is $(1-\mu) + \frac{1}{2}\mu = 1 - \frac{1}{2}\mu$. Given that $\mu \in [0,1]$, the winning probability is higher than $1/2$.

Now consider the case in which $b = 1$. Let ε be any number. $b_{i(2)}^1 - b_{i(1)}^1 < \varepsilon$ implies that $b_{i(1)}^1 = b_{i(2)}^1 = 1$. In this case, the probability that $i(2)$ wins is $1/2$. ∎

Lemma OA-1 *Consider the following differential equation*

$$y' = \frac{[1-\theta y][(1-\theta)^{n-1} - [1-\theta y]^{n-1}(2x-1)]}{(n-1)\theta x[(1-\theta y)^{n-1}(1-x) - (1-\theta)^{n-1}]}, \tag{OA-3}$$

with an initial condition $y(1/2 + (1-\theta)^{n-1}/2) = 0$. *There exists a solution* y *that is monotone increasing in range* $[1/2 + (1-\theta)^{n-1}/2, 1]$. *Moreover,* $y(1) \leqq 1$.

Proof. Consider the denominator of expression (OA-3),

$$D = (n-1)\theta x[(1-\theta y)^{n-1}(1-x) - (1-\theta)^{n-1}].$$

If $y = 0$, the square bracket term is zero if $x = 1 - (1-\theta)^{n-1}$. The square bracket is decreasing in both x and y. Hence, for any $x \in (1-(1-\theta)^{n-1}, 1]$ and $y \in [0,1]$, the square bracket term is strictly negative. Because $1-(1-\theta)^{n-1} < 1/2 + (1-\theta)^{n-1}/2$ when $\theta < 1 - \sqrt[n-1]{1/3}$, the right hand side of expression (OA-3) is Lipschitz continuous on $(x,y) \in [1/2 + (1-\theta)^{n-1}/2, 1] \times [0,1]$.

Now consider the numerator of expression (OA-3),

$$N = (1-\theta y)\left[(1-\theta)^{n-1} - (1-\theta y)^{n-1}(2x-1)\right].$$

Note that the term inside the square bracket is negative if

$$y \leqq \frac{1}{\theta}\left(1 - \frac{1-\theta}{\sqrt[n-1]{2x-1}}\right).$$

Hence,

$$\begin{aligned} sgn(N) &= sgn(1-\theta y) sgn\left(y - \frac{1}{\theta}\left(1 - \frac{1-\theta}{\sqrt[n-1]{2x-1}}\right)\right), \\ &= sgn(1-\theta y) sgn(y - f(x)), \\ \text{where } f(x) &= \frac{1}{\theta}\left(1 - \frac{1-\theta}{\sqrt[n-1]{2x-1}}\right). \end{aligned}$$

In the region $(x,y) \in [1/2+(1-\theta)^{n-1}/2, 1] \times [0,1]$, the right hand side of expression (OA-3) is positive below $y = f(x)$ and negative above it. Note that $f(1/2 + (1-\theta)^{n-1}/2) = 0$, $f(1) = 1$, and $f'(x) > 0$ for all $x \in (1/2 + (1-\theta)^{n-1}/2, 1)$ and $\theta \in (0,1)$. Figure OA.13 illustrates the phase diagram for the case of $\theta = 1/2$ and $n = 2$.

Given the Lipschitz continuity of the right hand side of expression (OA-3), there exists a local solution, $y(\cdot)$, to the initial condition problem. $y(\cdot)$ is increasing and $y \leqq f$. To see that $y(\cdot)$ can be extended to the interval $[1/2 + (1-\theta)^{n-1}/2, 1)$, suppose that it can only be extended to δ, where $\delta < 1$. Consider $y_\delta = \lim_{x\to\delta} y(x)$. Given that $y(\cdot)$ is monotone and bounded above by f, y_δ exists and is finite. Moreover, $y_\delta \leqq f(\delta)$. Given that the right hand side of expression (OA-3) is continuous, $\lim_{x\to\delta} y'(x)$ also exists and is finite. Now, consider solving for expression (OA-3) with an initial condition $(x,y) = (\delta, y_\delta)$. There exists a local solution, $y_\delta(\cdot)$, and, its value and derivative will agree with the original solution, i.e., $y_\delta(\delta) = \lim_{x\to\delta} y(x)$ and $\lim_{x\to\delta} y'(x) = y'_\delta(\delta)$. Hence, we get a contradiction. Finally, $y \leqq f$ implies $y(1) \leqq f(1) = 1$.

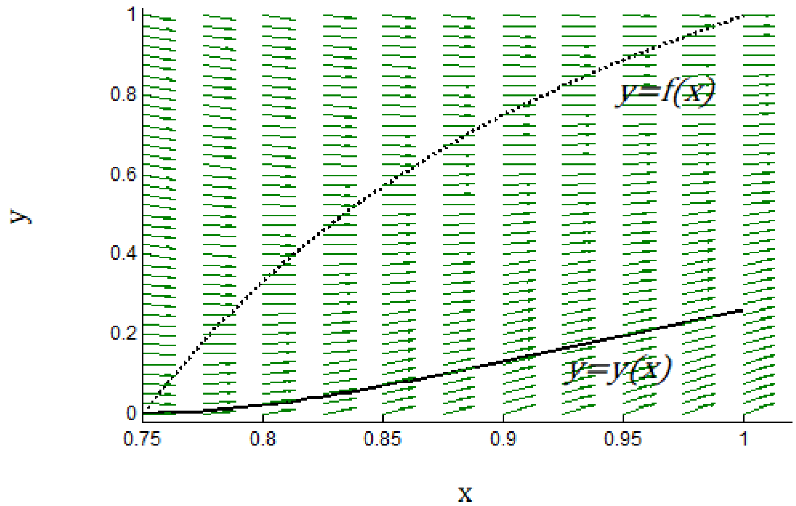


Figure OA.13: Phase Diagram of Differential Equation (OA-2) When $\theta = 1/2$ and $n = 2$. The solution to (OA-2) with initial value condition $H(1 - \theta/2) = 0$, is given by $y = y(x)$.

■

Online Appendix VII Robustness of the Test for Firm Best Response

In this section, we explore the robustness of our analysis in Section 5.1 [Optimality of Second-Round Bidding Strategy] to the assumption that bidder costs are private. Recall that the private value assumption guarantees that bidder i's third round bid is a valid upper bound on its perceived costs at the time of bidding in Round 2. In order to explore the sensitivity of our results to the assumption, we consider alternative bounds on bidder costs perceived at the time of bidding in Round 2 as follows:

$$c \leq (\mathfrak{b}^2 - \mathfrak{b}^3)\rho + \mathfrak{b}^3,$$

for different values of $\rho \in [0, 1]$. $\rho = 0$ corresponds to the analysis in Section 6.1. $\rho = 1$ corresponds to the extreme case in which bidders bid their perceived costs in Round 2.

The tables below show the results of the test presented in Table 4 for $\rho = 0.2, 0.5$, and 0.8, respectively. We find that the expected gains remain positive and statistically significant for the most part for $\rho = 0.2$ and 0.5. The results of the table show that as long

	$\Delta\pi_{i\|\mathcal{I}}$ (Yen)					
	$x = 99.0\%$	98.5%	98.0%	97.5%	97.0%	N
$\delta = 1\%$	734,221	1,088,161	1,171,978	1,094,705	919,117	4,499
	(138,239)	(211,864)	(253,542)	(283,027)	(308,768)	
3%	403,653	667,806	817,424	844,235	747,259	26,008
	(82,779)	(122,508)	(157,589)	(188,824)	(211,440)	
5%	300,131	519,350	661,845	700,832	626,052	42,141
	(59,344)	(92,502)	(123,096)	(149,432)	(169,165)	
15%	211,644	376,243	486,462	523,454	468,520	66,124
	(41,364)	(66,142)	(89,136)	(109,258)	(123,952)	

Note: Standard errors are computed using bootstrap and reported in parenthesis. All the numbers are in Yen.

Table OA.4: Expected Gain in Profits from Bidding $x\mathfrak{b}_i^2$: $\rho = 20\%$.

	$\Delta\pi_{i\|\mathcal{I}}$ (Yen)					
	$x = 99.0\%$	98.5%	98.0%	97.5%	97.0%	N
$\delta = 1\%$	253,285	345,541	249,705	38,303	-256,652	4,499
	(91,281)	(134,117)	(161,397)	(182,378)	(201,510)	
3%	164,188	247,219	222,801	90,410	-136,098	26,008
	(49,068)	(74,430)	(97,743)	(118,965)	(135,050)	
5%	113,514	184,270	172,109	63,970	-134,119	42,141
	(35,266)	(56,069)	(75,734)	(92,976)	(106,621)	
15%	70,765	123,525	113,172	28,453	-131,589	66,124
	(24,683)	(40,079)	(54,663)	(67,807)	(77,846)	

Note: Standard errors are computed using bootstrap and reported in parenthesis. All the numbers are in Yen.

Table OA.5: Expected Gain in Profits from Bidding $x\mathfrak{b}_i^2$: $\rho = 50\%$.

as bidder costs are bounded above by the average of its second round bid and its third round bid (i.e., $\rho = 0.5$) bidders can increase profits by bidding more aggressively.

	$\Delta\pi_{i\|\mathcal{I}}$ (Yen)					
	$x =$ 99.0%	98.5%	98.0%	97.5%	97.0%	N
$\delta = 1\%$	-157,259	-329,286	-607,374	-955,502	-1,372,424	4,499
	(64,431)	(79,271)	(94,009)	(108,654)	(124,037)	
3%	-59,214	-157,652	-356,453	-648,392	-1,004,778	26,008
	(25,329)	(37,556)	(51,097)	(64,603)	(77,043)	
5%	-55,751	-133,528	-300,411	-555,743	-877,208	42,141
	(17,231)	(26,101)	(35,822)	(45,949)	(55,572)	
15%	-50,483	-109,454	-240,271	-446,591	-711,634	66,124
	(11,802)	(18,043)	(25,051)	(32,713)	(40,090)	

Note: Standard errors are computed using bootstrap and reported in parenthesis. All the numbers are in Yen.

Table OA.6: Expected Gain in Profits from Bidding $x\mathfrak{b}_i^2$: $\rho = 80\%$.